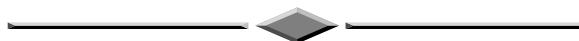


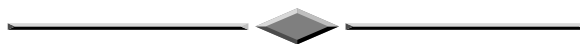
1. G. Dassios & P. Vafeas, “**Connection formulae for differential representations in Stokes flow**”, *Journal of Computational and Applied Mathematics (J. Comput. Appl. Math.)*, **133**, 283-294 (2001).

A pair of partial differential equations connecting the velocity with the total pressure field describes Stokes flow. Papkovich - Neuber and Boussinesq - Galerkin proposed two different differential representations of the flow fields (velocity and pressure) in terms of harmonic and biharmonic functions. On the other hand, spherical geometry provides the most widely used framework for representing small particles and obstacles embedded within a viscous, incompressible fluid characterizing the steady and non-axisymmetric Stokes flow. In the interest of producing ready-to-use basic functions for Stokes flow in spherical coordinates, we calculate the Papkovich - Neuber and the Boussinesq - Galerkin eigensolutions, generated by the well-known spherical harmonic and biharmonic eigenfunctions in the absence of singularities. Furthermore, connection formulae are obtained, by which we can transform any solution of the Stokes system from the Papkovich - Neuber to the Boussinesq - Galerkin eigenform and vice versa, only when the corresponding potentials have a particular form. We consider flows within internal and external domains with no singularity points on the axis of symmetry, where the two cases satisfy similar relations for the constant coefficients and their conjugates. Finally, we observe that the two general solutions are equivalent, but it is the differential representation Papkovich - Neuber that is given in a simpler form, as a result of the appearance of the biharmonic potential in the corresponding general solution proposed by Boussinesq - Galerkin.



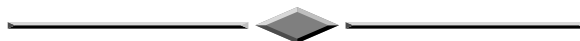
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2. P. Vafeas, “**On the connection between Stokes and Papkovich - Neuber spherical eigenfunctions in Stokes flow**”, *Bulletin of the Greek Mathematical Society (Bull. Greek Math. Soc.)*, **47**, 59-73 (2003).
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Stokes flow characterizes the steady and non-axisymmetric flow of an incompressible, viscous fluid at low Reynolds number and is described by a pair of partial differential equations connecting the velocity with the pressure field. Spherical geometry provides the most widely used framework for representing small particles embedded within a fluid that flows according to Stokes law and thus, the flow is assumed to be axisymmetric. The two different complete representations of the flow fields are considered here. The first one, named Stokes representation, is obtained, expressing the equation of motion in spherical coordinates, according to which the stream function is given in full series expansion in terms of separable eigenmodes. The second one, also valid in non-axisymmetric geometries, is the Papkovich - Neuber differential representation, where the flow fields are provided in terms of harmonic spherical eigenfunctions. In the interest of producing ready-to-use basic functions for axisymmetric Stokes flow in spherical coordinates by showing the different approach of solving such problems, we calculate the Stokes (2-D) and Papkovich - Neuber (3-D) eigensolutions, demonstrating the full series expansion. In the present work, connection formulae are obtained which relate the spherical harmonic eigenfunctions of the Papkovich - Neuber representation, considering rotational symmetry, with the separable spherical stream eigenfunctions, excluding singularities. In that way, we transform any solution of the Stokes symmetric system from one representation to the other taking advantage of each one, as the case may be.



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3. G. Dassios & P. Vafeas, “**Comparison of differential representations for radially symmetric Stokes flow**”, *Abstract and Applied Analysis (Abstr. Appl. Anal.)*, **4**, 347-360 (2004).
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The steady and creeping flow for incompressible viscous fluids is described by the well-known Stokes equations, connecting the biharmonic velocity with the harmonic total pressure field. Palaniappan - Nigam - Amaranath - Usha (PNAU) and Papkovich - Neuber (PN) proposed two different representations of the velocity and the pressure in terms of harmonic and biharmonic functions, which form a practical tool for many important physical applications. One of them is the particle-in-cell model for Stokes flow through a swarm of particles, which exhibits great theoretical and practical interest. Most of the analytical models in this realm consider spherical particles. Therefore, for many interior and exterior flow problems involving small particles, spherical geometry provides a very good approximation. In the interest of producing ready-to-use basic functions for Stokes flow in spherical coordinates, we calculate the PNAU and the PN eigensolutions, generated by the appropriate vector spherical harmonic, biharmonic eigenfunctions and the full series (no singularities) expansion is being demonstrated. Furthermore, connection formulae are obtained by which we can transform any solution of the Stokes system from the PN to the PNAU eigenform. We show that this procedure is not invertible since these formulae interrelate each PNAU potential with a specific combination of PN eigenfunctions, a fact that reflects the flexibility of the second representation. Hence, the advantage of the PN representation as it compares to the PNAU solution is established. This is demonstrated by solving the problem of the flow in a fluid cell filling the space between two concentric spherical surfaces with Kuwabara-type boundary conditions.



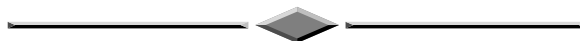
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4. G. Dassios, A.C. Payatakes & P. Vafeas, “**Interrelation between Papkovich - Neuber and Stokes general solutions of the Stokes equations in spheroidal geometry**”, *Quarterly Journal of Mechanics and Applied Mathematics (Quart. J. Mech. Appl. Math.)*, **57**, 181-203 (2004).
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Many practical applications involve particles (inorganic, organic, biological) with non-spherical but still axisymmetric shapes. The present work is concerned with some interesting aspects of the theoretical analysis of Stokes flow in spheroidal domains. Two different complete representations of Stokes flow are considered here. The first one is obtained through the theory of generalized eigenfunctions, according to which the stream function is expanded in terms of separable and *semiseparable* eigenfunctions. The second one, valid in non-axisymmetric geometries as well, is the Papkovich - Neuber differential representation, where the velocity and pressure fields are expressed in terms of harmonic spheroidal eigenfunctions. Connection formulae are obtained for the case of axisymmetric flows, which relate the spheroidal harmonic eigenfunctions of the Papkovich - Neuber representation with the semiseparable spheroidal stream eigenfunctions. In the case of axisymmetric spheroidal flows the Papkovich - Neuber approach is equivalent to the Stokes stream function approach, but the 3-D representation offers certain important advantages. Particle-in-cell models for Stokes flow through a swarm of particles are of substantial practical interest, because they provide a relatively simple platform for the analytical or semianalytical solution of heat and mass transport problems. The early versions of these models were concerned with spherical particles. For this reason particle-in-cell models for spheroidal particles were developed more recently. The flexibility of the Papkovich - Neuber differential representation is demonstrated by solving the problem of the flow in a fluid cell filling the space between two confocal spheroidal surfaces with Kuwabara-type boundary conditions.



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5. P. Vafeas, G. Perrusson & D. Lesselier, “**Low-frequency solution for a perfectly conducting sphere in a conductive medium with dipolar excitation**”, *Progress in Electromagnetics Research (Prog. Electromagn. Res.)*, **49**, 87-111 (2004).
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This contribution concerns the interaction of an arbitrarily orientated, time-harmonic, magnetic dipole with a perfectly conducting sphere embedded in a homogeneous conductive medium. A rigorous low-frequency expansion of the electromagnetic field in positive integral powers $(ik)^n$, k complex wavenumber of the exterior medium, is constructed. The first $n=0$ vector coefficient (static or Rayleigh) of the magnetic field is already available, so emphasis is on the calculation of the next two nontrivial vector coefficients (at $n=2$ and at $n=3$) of the magnetic field. Those are found in closed form from exact solutions of coupled (at $n=2$, to the one at $n=0$) or uncoupled (at $n=3$) vector Laplace equations. They are given in compact fashion, as infinite series expansions of vector spherical harmonics with scalar coefficients (for $n=2$). The good accuracy of both in-phase (the real part) and quadrature (the imaginary part) vector components of the diffusive magnetic field is illustrated by numerical computations in a realistic case of mineral exploration of the Earth by inductive means. This canonical representation, not available yet in the literature to this time (beyond the static term), may apply to other practical cases than this one in geoelectromagnetics, whilst it adds useful reference results to the already ample library of scattering by simple shapes using analytical methods.



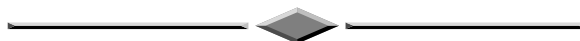
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6. A. Andrikopoulos & P. Vafeas, “**Maximal elements for binary relations on compact spaces**”, *Italian Journal of Pure and Applied Mathematics (Ital. J. Pure Appl. Math.)*, **19**, 85-90 (2006).
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The problem of characterizing the existence of maximal elements for preference relation has been stated firstly by Bergstrom in 1975. In this paper we give necessary and sufficient continuity conditions for the existence of maximal elements for a binary relation defined on a compact set.



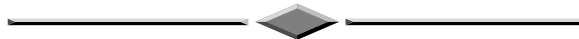
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7. G. Dassios & P. Vafeas, “**The 3D Happel model for complete isotropic Stokes flow**”, *International Journal of Mathematics and Mathematical Sciences (Int. J. Math. Math. Sci.)*, **46**, 2429-2441 (2004).
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Particle-in-cell models for the steady and non-axisymmetric flow of incompressible, viscous fluids at low Reynolds number (Stokes flow) are useful in the development of simple but reliable analytical expressions for swarms of particles. Most of the analytical models in this realm consider spherical particles. Therefore, spherical geometry provides the most widely used framework for representing small particles embedded within a fluid that flows according to Stokes description. Despite the fact that many physical applications involve particles with axisymmetric shapes leading to radially symmetric flows, it is of great theoretical and practical interest to investigate three-dimensional flow in assemblages of such particles. Here, the creeping flow through a swarm of spherical particles that move with constant velocity in an arbitrary direction and rotate with an arbitrary constant angular velocity in a quiescent Newtonian fluid is analyzed with a 3-D sphere-in-cell model. The mathematical treatment is based on the two concentric spheres model. The inner sphere comprises one of the particles in the swarm and the outer sphere consists of a fluid envelope. The appropriate boundary conditions of this non-axisymmetric formulation are similar to those of the 2-D sphere-in-cell Happel model, namely, non-slip flow condition on the surface of the solid sphere and nil normal velocity component and shear stress on the external spherical surface. The boundary value problem is solved with the aim of the complete Papkovitch - Neuber differential representation of the solutions for Stokes flow, which is valid in non-axisymmetric geometries and provides us with the velocity and total pressure fields in terms of harmonic spherical eigenfunctions. The solution of this three-dimensional model, which is self-sufficient in mechanical energy, is obtained in closed form and analytical expressions for the velocity, the total pressure, the angular velocity and the stress tensor fields are provided.



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8. P. Vafeas, “**Distribution of spheroidal focal singularities in Stokes flow**”, *International Journal of Pure and Applied Mathematics (Int. J. Pure Appl. Math.)*, **22**, 329-339 (2005).
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Stokes flow for the steady, non-axisymmetric motion of viscous, incompressible fluids in small Reynolds numbers (creeping flow), around small particles embedded within simply connected and bounded flow domains, is described by a pair of partial differential equations, which evolve the vector biharmonic velocity and the scalar harmonic total pressure fields. There exist many representations of the solutions of those kinds of flows, in three-dimensional domains, appearing in the form of differential operators acting on harmonic and biharmonic potentials. On the other hand, the development of Stokes theory for two-dimensional flows has the advantage that uses only one potential function (stream function) for the representation of the flow fields, but refers to axisymmetric flows. The effect of a distribution of sources - singularities, on the surface of a spheroidal particle or marginally on the focal segment, to the basic flow fields, is the goal of the present work. In particular, the proper confrontation of the problem is ensured by the introduction of the well-known Havelock's theorem for the presence of singularities, which provides us with the necessary integral representations of the velocity and the pressure. Moreover, the interrelation of the eigenforms of the Papkovitch - Neuber differential representation with those that arise from Stokes theory, in two-dimensional spheroidal geometry, completes the two manners of facing the problem.



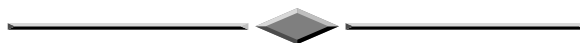
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9. P. Vafeas & G. Dassios, “**Stokes flow in ellipsoidal geometry**”, *Journal of Mathematical Physics (J. Math. Phys.)*, **47** (093102), 1-38 (2006).
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Particle-in-cell models for Stokes flow through a relatively homogeneous swarm of particles are of substantial practical interest, because they provide a relatively simple platform for the analytical or semianalytical solution of heat and mass transport problems. Despite the fact that many practical applications involve relatively small particles (inorganic, organic, biological) with axisymmetric shapes, the general consideration consists of rigid particles of arbitrary shape. The present work is concerned with some interesting aspects of the theoretical analysis of creeping flow in ellipsoidal, hence non-axisymmetric domains. More specifically, the low Reynolds number flow of a swarm of ellipsoidal particles in an otherwise quiescent Newtonian fluid, that move with constant uniform velocity in an arbitrary direction and rotate with an arbitrary constant angular velocity, is analyzed with an ellipsoid-in-cell model. The solid internal ellipsoid represents a particle of the swarm. The external ellipsoid contains the ellipsoidal particle and the amount of fluid required to match the fluid volume fraction of the swarm. The non-slip flow condition on the surface of the solid ellipsoid is supplemented by the boundary conditions on the external ellipsoidal surface which are similar to those of the sphere-in-cell model of Happel (self-sufficient in mechanical energy). This model requires zero normal velocity component and shear stress. The boundary value problem is solved with the aim of the potential representation theory. In particular, the Papkovitch - Neuber complete differential representation of Stokes flow, valid for non-axisymmetric geometries, is considered here, which provides the velocity and total pressure fields in terms of harmonic ellipsoidal eigenfunctions. The flexibility of the particular representation is demonstrated by imposing some conditions, which made the calculations possible. It turns out that the velocity of first degree, which represents the leading term of the series, is sufficient for most engineering applications, so long as the aspect ratios of the ellipsoids remains within moderate bounds. Analytical expressions for the leading terms of the velocity, the total pressure, the angular velocity and the stress tensor fields are obtained. Corresponding results for the prolate and the oblate spheroid, the needle and the disk, as well as for the sphere are recovered as degenerate cases. Novel relations concerning the ellipsoidal harmonics are included in appendix.



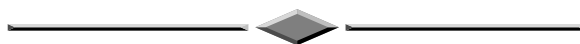
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10. V. Sevroglou & P. Vafeas, “**2D elastic scattering of a plane dyadic wave by a small rigid body and cavity**”, *ZAMM - Journal of Applied Mathematics and Mechanics (Z. Angew. Math. Mech.)*, **88**, 227-238 (2008).
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In this paper the scattering problem of the disturbance of a plane dyadic wave by a rigid body or a cavity, in two-dimensional elastodynamics is considered. The direct scattering problems are formulated in a dyadic form, and in each case, the corresponding longitudinal and transverse far-field scattering amplitudes are presented. We provide the necessary energy considerations and expressions for the differential and the scattering cross-section due to plane wave dyadic incidence. Next, the rigid body and cavity are considered to be small and are illuminated by a plane dyadic field. Finally, relative results for low-frequency scattering are obtained, and similar corresponding expressions for energy functionals in the far-field along with expressions for the differential and the total scattering cross-section are recovered as special cases.



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11. G. Dassios & P. Vafeas, “**On the spheroidal semiseparation for Stokes flow**”, *Research Letters in Physics (Res. Lett. Phys.)*, **2008 (135289)**, 1-4 (2008).
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Many heat and mass transport problems involve particle-fluid systems, where the assumption of the Stokes flow consideration provides a very good approximation for representing small particles embedded within a viscous, incompressible fluid characterizing the steady, creeping flow. The present work is concerned with some interesting practical aspects of the theoretical analysis of Stokes flow in spheroidal domains. The stream function ψ , for axisymmetric Stokes flow, satisfies the well-known equation $E^4\psi = 0$. Despite the fact that in spherical coordinates this equation admits separable solutions, this property is not preserved when one seeks solutions in the spheroidal geometry. Nevertheless, defining some kind of semiseparability, the complete solution for ψ in spheroidal coordinates has been obtained in the form of products combining Gegenbauer functions of different degree. Thus, the general solution is represented in a full series expansion in terms of eigenfunctions, which are elements of the space $\ker E^2$ (separable solutions) and in terms of generalized eigenfunctions, which are elements of the space $\ker E^4$ (semiseparable solutions). In this work we revisit this aspect by introducing a different and simpler way of representing the aforementioned generalized eigenfunctions. Consequently, additional semiseparable solutions are provided in terms of the Gegenbauer functions, whereas the completeness is preserved and the full series expansion is rewritten in terms of these functions.



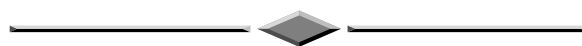
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12. P. Vafeas, G. Perrusson & D. Lesselier, “**Low-frequency scattering from perfectly conducting spheroidal bodies in a conductive medium with magnetic dipole excitation**”, *International Journal of Engineering Science (Int. J. Eng. Sci.)*, **47**, 372-390 (2009).
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Inductive electromagnetic means that are currently employed in the exploration of the Earth's subsurface and embedded voluminous bodies often call for an intensive use, primary at the modeling stage and later on at the inversion stage, of analytically demanding tools of field calculation. Under the aim of modeling implementation, this contribution is concerned with some interesting aspects of the low-frequency interaction of arbitrarily orientated (i.e. three-dimensional) time-harmonic magnetic dipoles, with 3-D perfectly conducting spheroidal bodies embedded in an otherwise homogeneous conductive medium. For many practical applications involving buried obstacles such as Earth's subsurface electromagnetic probing at low-frequency or any other physical cases (e.g. geoelectromagnetics), non-axisymmetric spheroidal geometry approximates sufficiently such kind of metallic shapes. On the other hand, our analytical approach deals with prolate spheroids, since the corresponding results for the oblate spheroidal geometry can be readily obtained through a simple transformation. The particular physical model concerns a solid impenetrable (metallic) body under a magnetic dipole excitation, where the scattering boundary value problem is attacked via rigorous low-frequency expansions for the incident, scattered and total electric and magnetic fields in terms of positive integral powers of (ik) , that is $(ik)^n$ for $n \geq 0$, where k stands for the complex wavenumber of the exterior medium. The purpose of the modeling is to contribute to a simple yet versatile tool to infer information on an unknown body from measurements of the three-component electric and magnetic fields nearby. Our goal is to obtain the most important terms of the low-frequency expansions of the electromagnetic fields, that is the static (for $n=0$) and the dynamic ($n=1,2,3$) terms. In particular, for $n=1$ there are no incident fields and thus no scattered ones, while for $n=0$ the Rayleigh electromagnetic expression is easily obtained in terms of infinite series. Emphasis is given on the calculation of the next two non-trivial terms (at $n=2$ and at $n=3$) of the aforementioned fields. Consequently, those are found in closed form from exact solutions of coupled (at $n=2$, to the one at $n=0$) or uncoupled (at $n=3$) Laplace equations and they are given in compact fashion, as infinite series expansions for $n=2$ or finite forms for $n=3$. Nevertheless, the difficulty of the Poisson's equation that has to be solved for $n=2$ is presented, whereas our analytical approach demands the use of the well-known cut-off method in order to obtain an analytical closed solution. Finally, this research adds useful reference results to the already ample library of scattering by simple shapes using analytical methods.



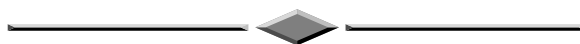
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13. P.M. Hatzikonstantinou & P. Vafeas “**A general theoretical model for the magnetohydrodynamic flow of micropolar magnetic fluids. Application to Stokes flow**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **33**, 233-248 (2010).
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Many practical applications, which have an inherent interest of physical and mathematical nature, involve the hydrodynamic flow in the presence of a magnetic field. Magnetic fluids comprise a novel class of engineering materials, where the coexistence of liquid and magnetic properties provides us with the opportunity to solve problems with high mathematical and technical complexity. Here, our purpose is to examine the micropolar magnetohydrodynamic flow of magnetic fluids by considering a colloidal suspension of ferromagnetic material (usually non-conductive) in a carrier magnetic liquid, which is in general electrically conductive. In this case, the ferromagnetic particles behave as rigid magnetic dipoles. Thus, the application of an external magnetic field, apart from the creation of an induced magnetic field of minor significance, will prevent the rotation of each particle, will increase the effective viscosity of the fluid and will cause the appearance of an additional magnetic pressure. Despite the fact that the general consideration consists of rigid particles of arbitrary shape, the assumption of spherical geometry is a very good approximation as a consequence of their small size. Our goal is to develop a general three-dimensional theoretical model that conforms to physical reality and at the same time permits the analytical investigation of the partial differential equations, which govern the micropolar hydrodynamic flow in such magnetic liquids. Furthermore, in the aim of establishing the consistency of our proposed model with the principles of both ferrohydrodynamics and magnetohydrodynamics, we take into account both magnetization and electrical conductivity of the fluid, respectively. Under this aspect, we perform an analytical treatment of these equations in order to obtain the three-dimensional effective viscosity and total pressure in terms of the velocity field, of the total (applied and induced) magnetic field and of the hydrodynamic and magnetic properties of the fluid, independently of the geometry of the flow. Moreover, we demonstrate the usefulness of our analytical approach by assuming a degenerate case of the aforementioned method, which is based on the reduction of the partial differential equations to a simpler shape that is similar to Stokes flow for the creeping motion of magnetic fluids. In view of this aim, we use the potential representation theory to construct a new complete and unique differential representation of magnetic Stokes flow, valid for non-axisymmetric geometries, which provides the velocity and total pressure fields in terms of easy-to-find potentials, via an analytical fashion.



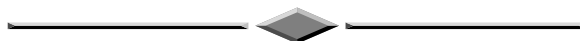
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14. G. Perrusson, P. Vafeas & D. Lesselier, “**Low-frequency dipolar excitation of a perfect ellipsoidal conductor**”, *Quarterly of Applied Mathematics (Quart. Appl. Math.)*, **68**, 513-536 (2010).
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This paper deals with the scattering by a perfectly conductive ellipsoid under magnetic dipolar excitation at low frequency. The source and the ellipsoid are embedded in an infinite homogeneous conducting ground. The main idea is to obtain an analytical solution of this scattering problem in order to have a fast numerical estimation of the scattered field that can be useful for real data inversion. Maxwell equations and boundary conditions, describing the problem, are firstly expanded using low-frequency expansion of the fields up to order three. It will be shown that fields have to be found incrementally. The static one (term of order zero) satisfies the Laplace equation. The next non-zero term (term of order two) is more complicated and satisfies the Poisson equation. The order-three term is independent of the previous ones and is described by the Laplace equation. They constitute three different scattering problems that are solved using the separated variables method in the ellipsoidal coordinate system. Solutions are written as expansions on the few analytically known scalar ellipsoidal harmonics. Details are given to explain how those solutions are achieved with an example of numerical results.



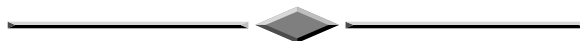
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15. P. Vafeas, P.K. Papadopoulos & P.M. Hatzikonstantinou, “**On the perturbation of the three-dimensional Stokes flow of micropolar fluids by a constant uniform magnetic field in a circular cylinder**”, *Mathematical Problems in Engineering (Math. Probl. Eng.)*, **2011 (659691)**, 1-41 (2011).
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Several practical applications in engineering technology involve the micropolar magnetohydrodynamic flow of magnetic fluids in the presence of a magnetic field. Here, we consider a colloidal suspension of non-conductive ferromagnetic material, which consists of very small spherical particles that behave as rigid magnetic dipoles, in a carrier liquid of approximately zero conductivity and low-Reynolds number properties. The interaction of a 3-D constant uniform magnetic field with the three-dimensional steady creeping motion (Stokes flow) of a viscous incompressible micropolar fluid in a circular cylinder is investigated, where the magnetization of the ferrofluid has been taken into account. We use a degenerate case of the general three-dimensional theoretical model that governs the micropolar hydrodynamic flow in such liquids, which is based on the reduction of the partial differential equations to a simpler shape that is similar to Stokes flow. Those magnetic Stokes equations contain the additional effective viscosity of the fluid due to the ferromagnetic particles in terms of the applied magnetic field and of both the hydrodynamic and magnetic properties of the fluid, independently of the geometry of the flow. Our goal is to apply the proper boundary conditions that conform to physical reality, so as to obtain the flow fields in a closed analytical form via the potential representation theory, and to study several characteristics of the flow. In view of this aim, we make use of an improved new complete and unique differential representation of magnetic Stokes flow, valid for non-axisymmetric geometries, which provides analytically the velocity and total pressure fields in terms of easy-to-find potentials. We use these results to simulate the creeping flow of a magnetic fluid inside a circular duct and to obtain the flow fields associated with this kind of flow.



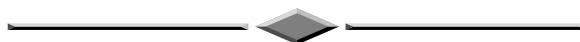
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16. P. Vafeas, P.K. Papadopoulos & D. Lesselier, “**Electromagnetic low-frequency dipolar excitation of two metal spheres in a conductive medium**”, *Journal of Applied Mathematics (J. Appl. Math.)*, **2012 (628261)**, 1-37 (2012).
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This work concerns the low-frequency interaction of a time-harmonic magnetic dipole, arbitrarily orientated in the three-dimensional space, with two perfectly conducting spheres embedded within a homogeneous conductive medium. In several applications, where two bodies are placed near one another, the 3-D bispherical geometry provides a good approximation. The particular physical problem is modeled by considering two solid impenetrable (metallic) obstacles, excited by a magnetic dipole, where the scattering boundary value problem is attacked via rigorous low-frequency expansions in terms of integral powers $(ik)^n$, where $n \geq 0$, k being the complex wave number of the exterior medium, for the incident, scattered and total electric and magnetic fields. We deal with the most important terms of the low-frequency expansions of the non-axisymmetric scattered electromagnetic fields, that is the static (for $n = 0$) and the dynamic ($n = 1, 2, 3$) terms, while for $n \geq 4$ the contribution of the additional terms is of minor significance. The calculation of the exact solutions, satisfying Laplace's and Poisson's differential equations, leads to infinite linear systems, solved approximately within any order of accuracy through a cut-off procedure and via numerical implementation. Thus, we obtain the electromagnetic fields in an analytically compact fashion as infinite series expansions of bispherical eigenfunctions. This particular electromagnetic scattering problem is then simulated in order to investigate the effect of the radii ratio, the relative position of the spheres and the position of the dipole on the real and imaginary parts of the calculated scattered magnetic field.



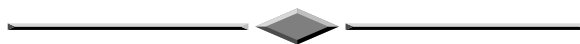
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17. F. Kariotou & P. Vafeas, “**The avascular tumour growth in the presence of inhomogeneous physical parameters imposed from a finite spherical nutritive environment**”, *International Journal of Differential Equations (Int. J. Differ. Equ.)*, **2012 (175434)**, 1-25 (2012).
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A well-known mathematical model of radially symmetric tumour growth is revisited in the present work. Under this aim a cancerous spherical mass lying in a finite concentric nutritive surrounding is considered. The host spherical shell provides the tumor with vital nutrients, receives the debris of the necrotic cancer cells and also transmits to the tumour the pressure imposed on its exterior boundary. We focus on studying the type of inhomogeneity that, the nutrient supply and the pressure field imposed on the host exterior boundary, can exhibit in order for the spherical structure to be supported. It turns out that if the imposed fields depart from being homogeneous, only a special type of interrelated inhomogeneity between nutrient and pressure can secure the spherical growth. The work includes an analytic derivation of the related boundary value problems based on physical conservation laws and their analytical treatment. Implementations in cases of special physical interest are examined and also existing homogeneous results from the literature are fully recovered.



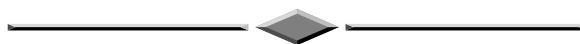
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18. P.K. Papadopoulos, P. Vafeas & P.M. Hatzikonstantinou, “**Ferrofluid pipe flow under the influence of the magnetic field of a cylindrical coil**”, *Physics of Fluids (Phys. Fluids)*, **24** (122002), 1-13 (2012).
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Ferrofluid pipe flow under the effect of a co-linear, finite length cylindrical coil is examined numerically. The specific flow configuration is chosen as it is encountered in engineering and bioengineering applications, such as magnetic drag targeting systems. The objective of the paper is twofold: Firstly, to investigate the accuracy of an analytical solution for the magnetization equation and assess its validity when used for non-uniform magnetic fields. It is found that it can be very helpful as a means of estimating the magnetization, especially for strong magnetic fields with low gradients. Secondly, to examine the effects of the magnetic field on the flow and study the relevant importance of the magnetic terms of the momentum equation. The parameters that we examine are the strength of the magnetic field and of its gradients, the volumetric concentration of the magnetic particles and the dimensions (length and diameter) of the coil. It is revealed that the axial pressure drop depends linearly on the volumetric concentration and that the magnetoviscosity effect is negligible in cases of non-uniform magnetic fields.



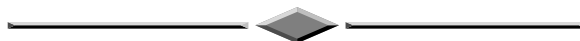
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19. G. Dassios, F. Kariotou & P. Vafeas, “**Invariant vector harmonics. The ellipsoidal case**”, *Journal of Mathematical Analysis and Applications (J. Math. Anal. Appl.)*, **405**, 652-660 (2013).
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We introduce a complete set of vector harmonic functions in an invariant form, that is, in a form that is independent of any coordinate system. In fact, we define three vector differential operators of the first order which, when they act on a scalar harmonic function they generate three independent vector harmonic functions. Then, we prove the relative independence properties and we investigate the characterization of every harmonic as irrotational or solenoidal field. We also prove that this set of functions forms a complete set of vector harmonics. Finally, we use these invariant expressions to recover the vector spherical harmonics of Hansen and to introduce vector ellipsoidal harmonics in \mathbb{R}^3 . Our method can be applied to any other coordinate system to produce the corresponding vector harmonics.



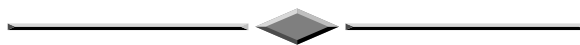
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20. K. Gazeli, P. Svarnas, P. Vafeas, P.K. Papadopoulos, A. Gkelios & F. Clément, **“Investigation on streamers propagating into a helium jet in air at atmospheric pressure: Electrical and optical emission analysis”**, *Journal of Applied Physics (J. Appl. Phys.)*, **114 (103304)**, 1-12 (2013).
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The plasma produced due to streamers guided by a dielectric tube and a helium jet in atmospheric air is herein studied electrically and optically. Helium streamers are produced inside the dielectric tube of a coaxial dielectric-barrier discharge and, upon exiting the tube, they propagate into the helium jet in air. The axisymmetric velocity field of the neutral helium gas while it penetrates the air is approximated with the PI-SO algorithm. At the present working conditions, turbulence helium flow is avoided. The system is driven by sinusoidal high voltage of variable amplitude (0-11 kV peak-to-peak) and frequency (5-20 kHz). It is clearly shown that a prerequisite for streamer development is a continuous flow of helium, independently of the sustainment or not of the dielectric-barrier discharge. A parametric study is carried out by scanning the range of the operating parameters of the system and the optimal operational window for the longest propagation path of the streamers in air is determined. For this optimum, the streamer current impulses and the spatiotemporal progress of the streamer UV-visible emission are recorded. The streamer mean propagation velocity is as well measured. The formation of copious reactive emissive species is then considered (in terms of intensity and rotational temperatures), and their evolution along the streamer propagation path is mapped. The main claims of the present work contribute to the better understanding of the physicochemical features of similar systems that are currently applied to various interdisciplinary engineering fields, including biomedicine and material processing.



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21. F. Kariotou & P. Vafeas, “**On the transversally isotropic pressure effect on avascular tumor growth**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **37**, 277-282 (2014).
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A typical mathematical formulation for describing avascular tumour growth is considered in the present work, in the frame of oblate spheroidal geometry focusing on the kind of exterior conditions under which such a geometrical structure is attainable. It turns out that given a transversally isotropic pressure field, an avascular tumour can exhibit an oblate spheroidal growth, only if the nutrient supply is provided in a specific form related to the pressure and following the tumour evolution. A geometrical reduction to the prolate geometry and recovering of existing results for the sphere is included.



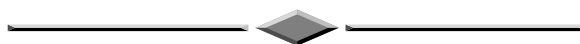
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22. F. Kariotou, P. Vafeas & P.K. Papadopoulos, “**Mathematical modeling of tumour growth in inhomogeneous spheroidal environment**”, *International Journal of Biology and Biomedical Engineering (Int. J. Biol. Biomed. Eng.)*, **8**, 132-141 (2014).
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Developing a mathematical model for cancer tumour growth that can be treated analytically and produce analytical results, is useful in the qualitative study of such complicated phenomenon. Most of such models consider radially symmetric tumours growing in homogeneous conditions, due to the availability of experimental data that concern mainly spherical tumours. Though, in vivo, the inhomogeneity of the host environment affects the geometrical features of the growing tumour mass, as shown in cases like the esophageal cancer. In the present work, we assume that the host tissue imposes the axisymmetric structure of a prolate spheroidal tumour via an appropriate pressure field and we investigate the evolution of such growth in a consistent nutritive microenvironment. To that purpose, the mathematical model that we consider consists of three boundary value problems, which describe the nutrient concentration, the inhibitor concentration and the pressure field in the interior and in the exterior of a layered prolate spheroid that models the tumour. These problems provide the necessary data for solving the evolution equation of the tumour's exterior boundary, which is a highly nonlinear ordinary differential equation. Additionally, our model exhibits a geometrical reduction to special cases and, mainly, to the spherical geometry in order to recover the existing results for the sphere.



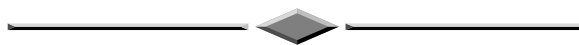
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23. P.K. Papadopoulos, P. Vafeas, P. Svarnas, K. Gazeli, P.M. Hatzikonstantinou, A. Gkelios & F. Clément, “**Interpretation of the gas flow field modification induced by guided streamer (‘plasma bullet’) propagation**”, *Journal of Physics D: Applied Physics (J. Phys. D: Appl. Phys.)*, **47 (425203)**, 1-16 (2014).
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Atmospheric-pressure non-equilibrium plasmas of noble gases in the form of “bullets” have attracted great attention, against cold low-pressure or thermal atmospheric-pressure plasmas, for multidisciplinary scientific fields such as material science and biomedicine, due to their unique compatible features. A key factor for the efficiency of most of these systems is the interaction between the noble-gas channel where the “bullets” (streamers) propagate and the plasma itself. It is the object of this article to demonstrate this interaction and to provide the explanation on the gas flow field modification induced by the plasma ignition. 3-D numerical model incorporating most of the governing equations, schlieren imaging and UV-visible high resolution optical emission spectroscopy are applied. In accordance with the present results, the mechanism leading to the flow field alteration is clearly related to the electrohydrodynamic force, while it is demonstrated that the gas temperature plays a minor role.



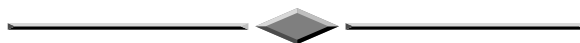
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24. P. Svarnas, P.K. Papadopoulos, P. Vafeas, A. Gkelios, F. Clément & A. Mavon, **“Influence of atmospheric pressure guided streamers (plasma bullets) on the working gas pattern in air”**, *IEEE Transactions on Plasma Science (IEEE Trans. Plasma Sci.)*, **42**, 2430-2431 (2014).
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This work is devoted to the study of gas flow fields related to helium atmospheric pressure guided streamer (plasma bullet) propagation in the air. For very weak up to moderate helium flows, the modification induced to the gas flow field by the plasma ignition is demonstrated; it is shown that the turbulent flow region is expanded and two conditions must be fulfilled regarding the working gas profile in the air for streamer propagation, i.e., laminar flow and high concentration in this laminar flow region.



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25. M. Doschoris & P. Vafeas, “**Connection formulae between ellipsoidal and spherical harmonics with applications to fluid dynamics and electromagnetic scattering**”, *Advances in Mathematical Physics (Adv. Math. Phys.)*, **2015 (572458)**, 1-12 (2015).
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The environment of the ellipsoidal system, significantly more complex than the spherical one, provides the necessary settings for tackling boundary value problems in anisotropic space. However, the theory of Lamé functions and ellipsoidal harmonics affiliated with the ellipsoidal system is rather complicated. A turning point would reside in the existence of expressions interlacing these two different systems. Still, there is no simple way, if at all, to bridge the gap. The present article addresses this issue. We provide explicit formulas of specific ellipsoidal harmonics expressed in terms of their counterparts in the classical spherical system. These expressions are then put into practice in the framework of physical applications.



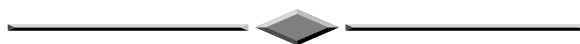
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26. P. Vafeas, P.K. Papadopoulos & P.M. Hatzikostas, “**Analytical integro-differential representation of flow fields for the micropolar Stokes flow of a conducting ferrofluid**”, *IMA Journal of Applied Mathematics (IMA J. Appl. Math.)*, **80**, 839-864 (2015).
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Practical physical applications of mathematical nature are frequently met in engineering technology and involve the low-Reynolds number flow of micropolar conducting fluids under the effect of magnetic fields. Here, we consider the 3-D creeping motion (Stokes flow), in steady state, of a non-conductive colloidal suspension of ferromagnetic material embedded within an electrically conductive, viscous and incompressible, carrier liquid. In such cases the ferromagnetic particles behave as rigid magnetic dipoles and react in the presence of an externally applied magnetic field, which is of general form and arbitrarily orientated in the three-dimensional space. Therein, an induced magnetic field of minor significance is created, while the effective viscosity of the fluid is increasing and an additional magnetic pressure is appeared. The consistency of the governing set of partial differential equations with the principles of both ferrohydrodynamics and magnetohydrodynamics is established by taking into account magnetization and electrical conductivity of the fluid, respectively. Our main intention is to use the potential representation theory to improve previous models and construct a new complete and unique integro-differential representation of the magnetic Stokes flow of conducting liquids, valid for any non-axisymmetric geometry, which provides the velocity and total pressure fields in a closed form and in terms of easy-to-find potentials, via a semi-analytical formalism. In order to demonstrate the usefulness of our analytical approach, we assume a degenerate case of the aforementioned method to simulate the creeping flow of a micropolar fluid with conductive properties inside a circular duct.



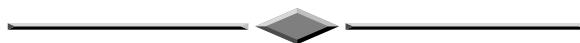
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27. G. Perrusson, P. Vafeas, I.K. Chatjigeorgiou & D. Lesselier, “**Low-frequency on-site identification of a highly conductive body buried in Earth from a model ellipsoid**”, *IMA Journal of Applied Mathematics (IMA J. Appl. Math.)*, **80**, 963-980 (2015).
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Identification of a highly-conductive orebody buried in Earth using an equivalent, perfectly-conducting, triaxial model ellipsoid is investigated. The real data available (three-component magnetic fields collected along a borehole due to a single-frequency current loop at the Earth surface) are simulated via a low-frequency, closed-form power series expansion of the electromagnetic fields scattered off an equivalent ellipsoid within a homogeneous, conductive medium, the source itself being idealized as a vertical magnetic dipole nearby. The approach provides formulations amenable to fast yet accurate computations, most of the work being in the construction of the formulations themselves, not in the numerical computations. The inversion scheme is described, which sees the iterative minimization of the least-square discrepancy between the fields due to a given ellipsoid and the data available. Unknowns are semi-axis lengths, angular orientations, and co-ordinates of its center. Numerical simulations illustrate the approach, before considering experimental single-well log data in a surface-to-borehole configuration at a mining site.



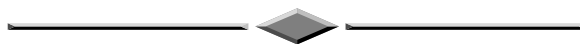
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28. P. Bakalis, P.M. Hatzikonstantinou & P. Vafeas, “**MFD formulations for the liquid metal flow in a curved pipe of circular cross section**”, *Computers & Fluids (Comput. Fluids)*, **119**, 1-12 (2015).
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The laminar fully developed magnetohydrodynamic (MHD) flow of a liquid metal into a curved pipe of circular cross section, subjected to a transverse external magnetic field, is studied. Three different formulations are used for the implementation of the electromagnetic variables. The extended Continuity Vorticity Pressure (CVP) numerical variational method for MHD flows is used for the coupling of the momentum and the continuity equation. Results are obtained for different values of the curvature (0-0.2) and of the Hartmann number (0-1000). The magnitude of the axial velocity is determined by the balance of the centrifugal and the electromagnetic forces. The results reveal the limits of applicability of the used electromagnetic models as the Hartmann number increases.



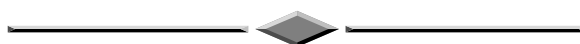
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29. P. Vafeas, D. Lesselier & F. Kariotou, “**Estimates for the low-frequency electromagnetic fields scattered by two adjacent metal spheres in a lossless medium**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **38**, 4210-4237 (2015).
-

Inductive electromagnetic means, currently employed in real physical applications and dealing with voluminous bodies embedded in lossless media, often call for analytically demanding tools of field calculation at modeling stage and later on at numerical stage. Here, one is considering two closely adjacent perfect conductors, possibly almost touching one another for which the 3-D bispherical geometry provides a good approximation. The particular scattering problem is modeled with respect to the two solid impenetrable metallic spheres, which are excited by a time-harmonic magnetic dipole, arbitrarily orientated in the three-dimensional space. The incident, the scattered and the total non-axisymmetric electromagnetic fields yield rigorous low-frequency expansions in terms of positive integral powers of the real-valued wave number in the exterior medium. We keep the most significant terms of the low-frequency regime, i.e. the static Rayleigh approximation and the first three dynamic terms, while the additional terms are small contributors and they are neglected. The typical Maxwell-type problem is transformed into intertwined either Laplace's or Poisson's potential-type boundary value problems with impenetrable boundary conditions. In particular, the fields are represented via 3-D infinite series expansions in terms of bispherical eigenfunctions, obtaining analytical closed-form solutions in a compact fashion. This procedure leads to infinite linear systems, which can be solved approximately within any order of accuracy through a cut-off technique.



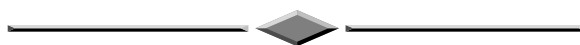
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30. M. Doschoris, G. Dassios, P. Vafeas, F. Kariotou & I.K. Chatjigeorgiou, “**Revisiting a numerical implementation of the EEG problem in ellipsoidal geometry**”, *Pioneer Journal of Advances in Applied Mathematics (Pioneer. J. Adv. Appl. Math.)*, **14**, 35-51 (2015).
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A triaxial ellipsoid provides an approximation of the average human brain which is much better than the broadly used spherical model. The analytical solution of the forward problem of the Electroencephalography (EEG) with an isolated dipolar source has been already derived and reported in the literature. This solution is expressed in terms of an eigenexpansion in ellipsoidal harmonics. Nevertheless, this expression was not possible to be handled effectively since no ellipsoidal harmonics of degree higher than seven were available in closed forms. In order to analyze further this problem an effective numerical algorithm has been developed, which generates the ellipsoidal harmonics of arbitrary degree and order in a numerical form. The algorithm has been compared with the known analytical eigenfunctions and the results manifested a perfect coincidence. Finally, this algorithm was used to construct a numerically stable solution of the electric potential on the surface of the head, which is generated by a single dipole of arbitrary position and orientation. The degree of ellipsoidal harmonics needed for a numerically convergent solution goes all the way up to 30 and the result provides a slightly improved model for tackling boundary value problems in ellipsoidal geometry.



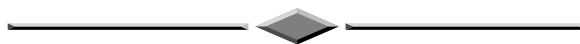
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31. P. Vafeas, P.K. Papadopoulos, P.-P. Ding & D. Lesselier, “**Mathematical and numerical analysis of low-frequency scattering from a PEC ring torus in a conductive medium**”, *Applied Mathematical Modelling (Appl. Math. Model.)*, **40**, 6477-6500 (2016).
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The electric and magnetic fields scattered off a non-penetrable ring torus, being characterized as perfect conductor, embedded in a homogeneous conductive medium and illuminated by a low-frequency magnetic dipole of arbitrary orientation and harmonic time-dependence are investigated herein. Upon definition of the complex wave number of the exterior medium k via its skin-depth, the 3-D scattering boundary value problem is handled via convenient low-frequency expansions in terms of powers of $(ik)^n$, $n \geq 0$ for the fields. A Maxwell-type problem is transformed into intertwined Laplace's or Poisson's potential-type boundary value problems with impenetrable boundary conditions. Using a toroidal coordinate system attached to the torus, they are solved as infinite series expansions for the fields in terms of toroidal eigenfunctions. In practice, what is accessible to the measurement is the scattered magnetic field. The static term ($n=0$) provides most of its real (or in-phase) part and the second-order term ($n=2$) consists of most of its imaginary part (quadrature), where in both cases a small contribution of the third-order term ($n=3$) is being calculated. For $n=1$, there exists no field, while the terms for $n \geq 4$ and for such kind of applications, have been proved to be of minor significance, hence they are neglected. The resulting infinite linear systems can be solved at any accuracy level through a cut-off process or via an analytical technique based on the method of finite continuous fraction solutions. Basics of the far-field approximation and the magnetic polarisability tensor are also included. At implementation stage, simulations are proposed in various situations, where a full-wave, finite-element approach is discussed.



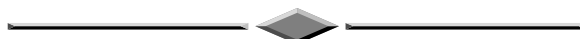
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32. P. Vafeas, “**Low-frequency electromagnetic scattering by a metal torus in a lossless medium with magnetic dipolar illumination**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **39**, 4268-4292 (2016).
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The present contribution is concerned with an analytical presentation of the low-frequency electromagnetic fields, which are scattered off a highly conductive ring torus that is embedded within an otherwise lossless ambient and interacting with a time-harmonic magnetic dipole of arbitrary orientation, located nearby in the three-dimensional space. Therein, the particular 3-D scattering boundary value problem is modeled with respect to the solid impenetrable torus-shaped body, where the toroidal geometry fits perfectly. The incident, the scattered and the total non-axisymmetric magnetic and electric fields are expanded in terms of positive integral powers of the real-valued wave number of the exterior medium at the low-frequency regime, whereas the static Rayleigh approximation and the first three dynamic terms provide the most significant part of the solution, since all the additional terms are small contributors and, hence, they are neglected. Consequently, the typical Maxwell-type physical problem is transformed into intertwined either Laplace's or Poisson's potential-type boundary value problems with the proper conditions, attached to the metallic surface of the torus. The fields of interest assume representations via infinite series expansions in terms of standard toroidal eigenfunctions, obtaining in that way analytical closed-form solutions in a compact fashion. Although this mathematical procedure leads to infinite linear systems for every single case, these can be readily and approximately solved at a certain level of desired accuracy through standard cut-off techniques.



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33. D.K. Logothetis, P.K. Papadopoulos, P. Svarnas & P. Vafeas, “**Numerical simulation of the interaction between helium jet flow and an atmospheric-pressure “plasma jet”**”, *Computers & Fluids (Comput. Fluids)*, **140**, 11-18 (2016).
-

In this work we study the interaction of an atmospheric-pressure “plasma jet” with the hydrodynamic flow of the working gas. The study is based on the comparison between numerical simulation results and experimental data collected from the literature. Plasma reactors of three different configurations are considered, using a simple model, which focuses on the electro-hydrodynamic force importance. The objective is to evaluate the ability of the model to capture the resulting interaction between the “plasma jet” and the working gas for different reactor configurations. It is also aimed to find out possible correlations between the main parameters of the system, which may be useful for theoretical model development and reactor improved designing. In the context of the present model, it is assumed that the local electro-hydrodynamic force can be expressed via the product of a constant-motive part, which depends on the plasma setup and parameters, with the working gas local concentration, which expresses the dependence of the ionization rate on the gas concentration. The simulation results unveil that the constant-motive part is independent of the flow rate and inversely proportional to the diameter of the dielectric tube of the plasma reactor.



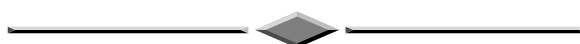
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34. J.C.-E. Sten, G. Fragoyiannis, P. Vafeas, P. Koivisto & G. Dassios, “**Theoretical development of elliptic cross-sectional hyperboloidal harmonics and their application to electrostatics**”, *Journal of Mathematical Physics (J. Math. Phys.)*, **58 (053505)**, 1-19 (2017).
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The analytic computation of electric and magnetic fields near corners and edges is important in many applications related to science and engineering. However, such complicated situations are hard to deal with, since they accumulate charges and consequently they mathematically represent singularities. In order to model this singular behavior, we introduce a novel method, which is related to the geometry and the analysis of the ellipsoidal coordinate system. Indeed, adopting the benefits of the corresponding coordinate surfaces, we use a general non-circular double cone, being the asymptote of a two-sided hyperboloid of two sheets with elliptic cross-section, which matches almost perfectly the particular physics and captures the corresponding essential features in a fully three-dimensional fashion. To this end, our analytical technique employs the ellipsoidal geometry and adapts the ellipsoidal functions (solutions of the well-known Lamé equation) so as to construct a new set of the so-called elliptic cross-sectional hyperboloidal harmonics, supplemented by the appropriate orthogonality rules on every constant coordinate surface. By first recollecting the key results of the coordinate system and the related potential functions, including the indispensable orthogonality results, we demonstrate our method to the solution of two boundary value problems in electrostatics. Both refer to a non-penetrable two-hyperboloid of elliptic cross-section and its double-cone limit, the first one being charged and the second one scattering off a plane wave. Closed form expressions are derived for the related fields, while the already known formulae from the literature are readily recovered, all cases being followed by the appropriate numerical implementation.



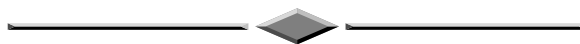
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35. P. Vafeas, “**Revisiting the low-frequency dipolar perturbation by an impenetrable ellipsoid in a conductive surrounding**”, *Mathematical Problems in Engineering (Math. Probl. Eng.)*, **2017 (9420658)**, 1-16 (2017).
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Real physical applications concerning underground detections or other identifications of voluminous bodies require analytically demanding tools of field calculation at modeling and numerical stage. This contribution deals with the scattering by a metallic ellipsoidal target, embedded in a homogeneous conductive medium, which is stimulated when a 3-D time-harmonic magnetic dipole is operating nearby at low frequency. The incident, the scattered and the total three-dimensional electromagnetic fields, which satisfy Maxwell's equations, yield low-frequency expansions in terms of positive integral powers of the complex-valued wave number of the exterior medium. We preserve the most significant terms of the low-frequency realm, those being the static Rayleigh approximation and the first three dynamic terms, while the additional terms are neglected, since their contribution is minor. The Maxwell-type problem is transformed into intertwined either Laplace's or Poisson's potential-type boundary value problems with impenetrable boundary conditions. On the other hand, the environment of a genuine ellipsoidal coordinate system provides the necessary setting for tackling such problems in anisotropic space. The fields are represented via non-axisymmetric infinite series expansions in terms of harmonic eigenfunctions, affiliated with the ellipsoidal system, obtaining analytical closed-form solutions in a compact fashion. Until nowadays, such problems were attacked by using the very few ellipsoidal harmonics exhibiting an analytical form. In the present article, we address this issue by incorporating the full series expansion of the potentials and utilizing the entire subspace of ellipsoidal harmonic eigenfunctions. That way, it is feasible to introduce any numerical technique for implementing the evaluated fields up to very high orders of harmonics, depending on the desired accuracy for the convergence of the potential series.



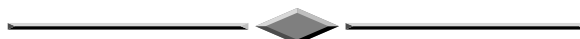
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36. P. Vafeas, “**On the integro-differential general solution for the unsteady micropolar Stokes flow of a conducting ferrofluid**”, *Quarterly of Applied Mathematics* (*Quart. Appl. Math.*), **76**, 19-37 (2018).
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The three-dimensional (3-D) unsteady creeping motion, corresponding to Stokes flow, of a non-conductive colloidal suspension of ferromagnetic particles, which are embedded within an otherwise electrically conducting, viscous and incompressible, carrier liquid, is considered in this contribution. This group of micropolar conducting ferrofluids comprises a novel class of engineering materials that respond in the presence of a general externally applied magnetic field, which is arbitrarily orientated in the three-dimensional domain of practical interest. Therein, an induced magnetic field of minor importance is created, while the effective viscosity of the fluid is increasing and an additional magnetic pressure is appeared. In order to be compatible with the principles of both ferrohydrodynamics and magnetohydrodynamics, we readily include the magnetization and the electrical conductivity of the magnetic fluid, respectively into the governing partial differential equations of the particular physical system. Employing the potential representation theory, we fabricate a new integro-differential general solution for the situation under investigation, which provides the time-dependent velocity and total pressure fields in a 3-D spaced closed form and in terms of easy-to-find potentials, via a semi-analytical shape. This generalized representation is proved to be complete, whilst it is valid for any non-axisymmetric geometry. We demonstrate the applicability of our analytical approach, by introducing a basic degenerate case of the aforementioned method to simulate the time-dependent creeping flow of a micropolar fluid with conductive properties inside a circular duct.



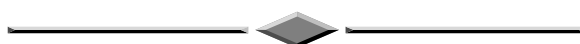
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37. M. Doschoris, P. Vafeas & G. Fragoyiannis “**The influence of surface deformations on the forward magnetoencephalographic problem**”, *SIAM Journal on Applied Mathematics (SIAM J. Appl. Math.)*, **78**, 963-976 (2018).
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A perturbational model is developed providing explicit computationally efficient solutions for the forward magnetoencephalographic problem, namely calculating the external magnetic fields for known neuronal sources. The aim of the study is to investigate the sensitivity of the particular measurements to deformations occurring on the conductor's surface. These geometric variations represent irregularities in head shapes and correspond to two major situations: (1) Localized acquired injuries of the scalp-skull delivered by external forces; (2) Craniofacial alterations due to natural mechanisms or defects. The presented methodology has the following advantages. Firstly, it supports the installation of tailored functions, which individually describe aforesaid deformations. Secondly, it allows rapid calculation of the forward problem for superficial cerebral activity, where similar numerical methods produce large errors. Our results indicate that surface deformations can have an eminent impact on magnetoencephalographic measurements under the condition that the neuronal brain activity is located beneath the deformed area, as well as on the extent of the deformation itself. In situations where surface deformations are not taken into account, the error made, varying between 5 to 25 per cent, propagates with distance.



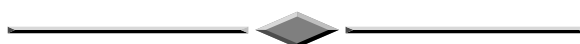
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38. P. Vafeas, “**Dipolar excitation of a perfectly electrically conducting spheroid in a lossless medium at the low-frequency regime**”, *Advances in Mathematical Physics (Adv. Math. Phys.)*, **2018 (9587972)**, 1-20 (2018).
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The electromagnetic vector fields, which are scattered off a highly conductive spheroid that is embedded within an otherwise lossless medium, are investigated in this contribution. A time-harmonic magnetic dipolar source, located nearby and operating at low frequencies, serves as the excitation primary field, being arbitrarily orientated in the three-dimensional space. The main idea is to obtain an analytical solution of this scattering problem, using the appropriate system of spheroidal coordinates, such as a possibly fast numerical estimation of the scattered fields could be useful for real data inversion. To this end, incident and scattered, as well total fields are written in a rigorous low-frequency manner in terms of positive integral powers of the real-valued wave number of the exterior environment. Then, the Maxwell-type problem is converted to interconnected either Laplace's or Poisson's equations, complemented by the perfectly conducting boundary conditions on the spheroidal object and the necessary radiation behavior at infinity. The static approximation and the three first dynamic contributors are sufficient for the present study, while terms of higher orders are neglected at the low-frequency regime. Henceforth, the 3-D scattering boundary value problems are solved incrementally, whereas the determination of the unknown constant coefficients leads either to concrete expressions or to infinite linear algebraic systems, which can be readily solved by implementing standard cut-off techniques. The non-axisymmetric scattered magnetic and electric fields follow and they are obtained in an analytical compact fashion via infinite series expansions in spheroidal eigenfunctions. In order to demonstrate the efficiency of our analytical approach, the results are degenerated so as to recover the spherical case, which validates this approach.



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39. P. Svarnas, P.K. Papadopoulos, D. Athanasopoulos, K. Sklias, K. Gazeli & P. Vafeas, “**Parametric study of thermal effects in a capillary dielectric-barrier discharge related to plasma jet production: Experiments and numerical modelling**”, *Journal of Applied Physics (J. Appl. Phys.)*, **124** (064902), 1-13 (2018).
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In the present work, a capillary dielectric-barrier discharge of coaxial electrode configuration, commonly employed to atmospheric-pressure cold plasma jet production, is studied in terms of thermal effects. The discharge is driven by sinusoidal high voltage in the kHz range and operates with helium gas channeled into a capillary dielectric tube having one end opened to the atmospheric air. The voltage amplitude and frequency, gas flow rate and discharge volume are varied independently, and thermal effects are investigated by experimentally acquired results coupled with numerically determined data. The experiments refer to electrical power and time-resolved temperature measurements and high resolution optical emission spectroscopy. The numerical modeling incorporates an electro-hydrodynamic force model in the governing equations to take into account the helium-air interplay and uses conjugate heat transfer analysis. The comparison between experimental and numerical data shows that power is principally consumed on the dielectric barrier and the gas phase reactions. A linear relation between steady state temperatures and supplied power, independently of the designing and operating conditions, is experimentally established. However, gas flow rate affects differently the thermal effects compared to the other parameters, supporting the idea of a two-fold nature of these systems, i.e. electrical and hydrodynamic. The main claim states the possibility of correlating designing and operating parameters for evaluating heat distribution and gas temperature trends in capillary dielectric-barrier discharges used for plasma jet production. This is of high importance for processing temperature-sensitive materials, including bio-specimens.



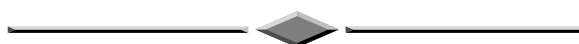
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40. P. Vafeas, A. Skarlatos, T. Theodoulidis & D. Lesselier, “**Semi-analytical method for the identification of inclusions by air-cored coil interaction in ferromagnetic media**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **41**, 6422-6442 (2018).
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The magnetostatic harmonic fields scattered by a near-surface air inclusion of arbitrary shape, embedded in a conductive ferromagnetic medium and illuminated by a current-carrying coil, are investigated. The scattering domain is separated into homogeneous subdomains under the assumption of a suitable truncation at a long distance from the incident source, whereas a perfect magnetic boundary condition is implied. The introduced methodology addresses the full coupling between the two interfaces, i.e. the plane that distinguishes the half-space ferromagnetic material from the open air and the arbitrary surface among the inclusion and the ferromagnetic region. Therein, continuity conditions are applied in a rigorous way, while the expected behavior of the fields, either as ascending or as descending, are taken into account. The potentials associated with the half-space are expanded via cylindrical harmonic eigenfunctions, while those related with the inclusion's arbitrary geometry admit generalized-type formalism. However, since the transmission conditions involve potentials with different eigenexpansions, we are obliged to rewrite cylindrical to generalized functions and vice versa, obtaining handy relationships in terms of easy-to-handle integrals, where orthogonality then is feasible. Once done, the calculation of the exact solutions leads to infinite linear algebraic systems, manipulated through standard cut-off techniques. Thus, we obtain the implicated fields in a general analytical and compact fashion, independent of the inclusion's geometry. We demonstrate the efficiency of the analytical model approach, assuming the degenerate case of a spherical inclusion, whereas the air-cored coil simulation via a numerical procedure validates our method. The calculation is very fast, rendering it suitable for use with parametric inversion algorithms.



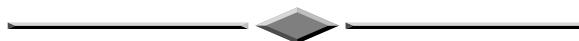
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41. G. Gavriil, P. Vafeas, A. Kanavouras & F.A. Coutelieris, “**Validation method for the systematization of results based on a similarity concept**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **42**, 656-666 (2019).
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The development of a functional methodological approach is presented here, to clarify a globally valid way of evaluating the precision of mathematical modeling of physical and/or chemical processes. Starting from the description of the system, a phenomenon accompanied by a disclaiming hypothesis is investigated, against which the knowledge is accumulated with time. Moreover, the possibility of the evolution of any phenomenon being interrupted when a parameter overpasses a critical threshold, after which the hypothesis is not any more valid, is introduced. This possibility should be obtained through the dependence of a selected macroscopic quantity (marker) on a specific parameter. To apply this methodology, the problem of Stokes flow through a granular medium of spheroidal grains has been selected as an indicative case study. The prolate spheroidal configuration is considered, since the results for the oblate spheroid can be recovered via simple transformation. Therein, the three-dimensional flow fields are initially constructed analytically via the Papkovitch - Neuber differential representation, which provides the velocity and pressure fields in terms of harmonic spheroidal eigenfunctions. Next, under the Kuwabara-type spheroidal 2-D unit cell concept, the above expressions degenerate to the axisymmetric case and the full solution is then obtained, keeping the leading terms of the series, which are adequate for most engineering applications for specific aspect ratio of the spheroids. In the sequel, the aforementioned problem is solved numerically for a 3-D extension of the same model, where this numerical solution has been achieved by using the Finite Volumes Method (FVM), while the resulting linear systems were approximated by applying the well-known Successful Over-Relaxation (SOR) concept. Finally, outcomes by both models have been compared via the above methodology, resulting to objective and reliable accuracy criteria.



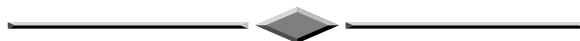
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42. P. Papadopoulos, D. Athanasopoulos, K. Sklias, P. Svarnas, N. Mourousias, K. Vratsinis & P. Vafeas, “**Generic residual charge based model for the interpretation of the electrohydrodynamic effects in cold atmospheric pressure plasmas**”, *Plasma Sources Science and Technology (Plasma Sources Sci. Technol.)*, **28 (065005)**, 1-17 (2019).
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In Cold Atmospheric Pressure Plasmas (CAPPs), the residual charge that exists in the wake of the streamers plays an important role in the acceleration of the working gas. This paper presents a model that links the drift of the net residual ionic charge density, under the effect of the local electric field, with the momentum increase of the gas. In the model, the ions and the neutrals are considered as separate phases and the conservation equations for the two phases are connected via the ionic pressure. The residual charge density is quantified through an approximate approach that considers the streamer events to be “instantaneous”, in order to avoid the excessive computational cost of resolving the propagation of each streamer. For the validation of the residual charge model with the “instantaneous” streamer approach, comparisons are made with experimental data from three plasma jet reactors. The electrode configuration of the reactors and the varied parameters (applied voltage, gas flow rate) are chosen so as to cover a broad range of different cases, in order to assess the generality of the model. The comparisons concern the gas flow and visible plasma patterns. It is found that the numerically simulated flow structures are in agreement with the corresponding schlieren images and that the residual charge density is a fair indicator of the visible plasma channel.



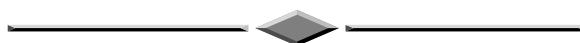
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43. P. Vafeas, P. Bakalis & P.K. Papadopoulos, “**Effect of the magnetic field on the ferrofluid flow in a curved cylindrical annular duct**”, *Physics of Fluids (Phys. Fluids)*, **31 (117105)**, 1-15 (2019).
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The laminar fully developed ferrofluid flow of an otherwise magnetic fluid into a curved annular duct of circular cross-section, subjected to a transverse external magnetic field, is studied in the present work. The specific geometry is chosen as it is encountered in heat exchangers and mixers, where compactness is a priority. Results are obtained for different values of curvature, field strength and particles' volumetric concentration. A computational algorithm is used which couples the continuity, Navier-Stokes and magnetization equations, using a non-uniform grid. The velocity - pressure coupling is achieved using the so-called Continuity-Vorticity-Pressure (C.V.P.) variational equations method, adapted to the toroidal-poloidal coordinate system. The results confirm the ability of the method to produce accurate results in curvilinear coordinates and stretched grids, which is important for the standardization of the method's application to generalized coordinate systems. Concerning the micropolar flow characteristics, the results reveal the effect of the magnetic field on the ferrofluid flow. It is shown that the axial velocity distribution is highly affected by the field strength and the volumetric concentration, that the axial pressure drop depends almost linearly on the field strength and that a secondary flow is generated due to the combined effect of the external magnetic field and the curvature. The present analysis provides important insight into the effect of the three main parameters, revealing cases where a straight annular pipe might be preferable to a curved one and specific parts of the pipe that could be susceptible to enhanced loads, giving information that is crucial for design optimization.



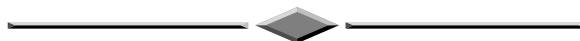
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44. G. Fragoyiannis, F. Kariotou & P. Vafeas, “**On the avascular ellipsoidal tumour growth model within a nutritive environment**”, *European Journal of Applied Mathematics (Eur. J. Appl. Math.)*, **31**, 111-142 (2020).
-

The present work is part of a series of studies conducted by the authors on analytical models of avascular tumour growth that exhibit both geometrical anisotropy and physical inhomogeneity. In particular, we consider a tumour structure formed in distinct ellipsoidal regions occupied by cell populations at a certain stage of their biological cycle. The cancer cells receive nutrient by diffusion from an inhomogeneous supply and they are subject to an also inhomogeneous pressure field imposed by the tumour microenvironment. It is proved that the lack of symmetry is strongly connected to a special condition that should hold between the data imposed by the tumour's surrounding, in order for the ellipsoidal growth to be realizable, a feature already present in other non-symmetrical yet more degenerate models. The nutrient and the inhibitor concentration, as well as the pressure field are provided in analytical fashion via closed form series solutions in terms of ellipsoidal eigenfunctions, while their behavior is demonstrated by indicative plots. The evolution equation of all the tumour's ellipsoidal interfaces is postulated in ellipsoidal terms and a numerical implementation is provided in view of its solution. From the mathematical point of view, the ellipsoidal system is the most general coordinate system that the Laplace operator, which dominates the mathematical models of avascular growth, enjoys spectral decomposition. Therefore, we consider the ellipsoidal model presented in this work, as the most general analytic model describing the avascular growth in inhomogeneous environment. Additionally, due to the intrinsic degrees of freedom inherited to the model by the ellipsoidal geometry, the ellipsoidal model presented can be adapted to a very populous class of avascular tumours, varying in figure and in orientation.



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45. P. Vafeas, P.K. Papadopoulos, G.P. Vafakos, P. Svarnas & M. Doschoris, “**Modelling the electric field in reactors yielding cold atmospheric-pressure plasma jets**”, *Scientific Reports (Sci. Rep.)*, **10 (5694)**, 1-15 (2020).
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The behavior of the electric field in Cold Atmospheric-Pressure Plasma jets (CAPP jets) is important in many applications related to fundamental science and engineering, since it provides crucial information related to the characteristics of plasma. To this end, this study is focused on the analytic computation of the electric field in a standard plasma reactor system (in the absence of any space charge), considering the two principal configurations of either one-electrode or two-electrodes around a dielectric tube. The latter is considered of minor contribution to the field calculation that embodies the working gas, being an assumption for the current research. Our analytical technique employs the cylindrical geometry, properly adjusted to the plasma jet system, whereas handy subdomains separate the area of electric activity. Henceforth, we adapt the classical Maxwell's potential theory for the calculation of the electric field, wherein standard Laplace's equations are solved, supplemented by the appropriate boundary conditions and the limiting conduct at the exit of the nozzle. The theoretical approach matches the expected physics and captures the corresponding essential features in a fully three-dimensional fashion via the derivation of closed-form expressions for the related electrostatic fields as infinite series expansions of cylindrical harmonic eigenfunctions. The feasibility of our method for both cases of the described experimental setup is eventually demonstrated by efficiently incorporating the necessary numerical implementation of the obtained formulae. The analytical model is benchmarked against reported numerical results, whereas discrepancies are commented and prospective work is discussed.



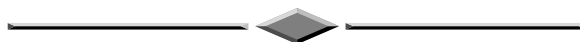
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46. P. Vafeas, “**Low-frequency dipolar electromagnetic scattering by a solid ellipsoid in lossless environment**”, *Studies in Applied Mathematics (Stud. Appl. Math.)*, **145**, 217-246 (2020).
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Electromagnetic wave scattering phenomena for target identification are important in many applications related to fundamental science and engineering. Here, we present an analytical formulation for the calculation of the magnetic and electric fields that scatter off a highly conductive ellipsoidal body, located within an otherwise homogeneous and isotropic lossless medium. The primary excitation source assumes a time-harmonic magnetic dipole, precisely fixed and arbitrarily orientated that operates at low frequencies and produces the incident fields. The scattering problem itself is modeled with respect to rigorous expansions of the electromagnetic fields at the low-frequency regime in terms of positive integral powers of the real wave number of the ambient. Obviously, the Rayleigh static term and a few dynamic terms are sufficient for the purpose of the present work, since the additional terms are neglected due to their minor contribution. Therein, the classical Maxwell's theory is suitably modified, leading to intertwined either Laplace's or Poisson's equations, accompanied by the impenetrable boundary conditions for the total fields and the limiting behavior at infinity. On the other hand, the complete spatial anisotropy of the three-dimensional space is secured via the introduction of the genuine ellipsoidal coordinate system, being appropriate for tackling incrementally such scattering boundary value problems. The non-axisymmetric fields are obtained via infinite series expansions in terms of ellipsoidal harmonic eigenfunctions, providing handy closed-form solutions in a compact fashion, whose validity is verified by a straightforward reduction to simpler geometries of the metal object. The main idea is to demonstrate an efficient methodology, according to which the constructed analytical formulae can offer the appropriate environment for a fast numerical estimation of the scattered electromagnetic fields that could be useful for real data inversion.



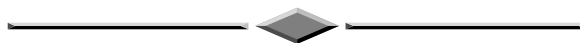
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47. P. Vafeas, J.C.-E. Sten & I.K. Chatjigeorgiou, “**On the electrostatic potential for the two-hyperboloid and double-cone of a single sheet with elliptic cross-section**”, *Quarterly Journal of Mechanics and Applied Mathematics (Quart. J. Mech. Appl. Math.)*, **74**, 117-135 (2021).
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The study of the response of divergence-free electric fields near corners and edges, resembling singularities that accumulate charges, is significant in modern engineering technology. A sharp point can mathematically be modeled with respect to the tip of the one sheet of a double-cone. Here, we investigate the behavior of the generated harmonic potential function close to the apex of a single-sheeted two-hyperboloid with elliptic cross-section, whose asymptote is the corresponding elliptic double-cone with one sheet present. Hence, the electrostatic potential problem, involving a single sheet of a two-hyperboloid, is developed using the theory of ellipsoidal-hyperboloidal harmonics, wherein the particular consideration enforces as solution in terms of generalized Lamé functions of non-integer order. A numerical method to determine these functions is outlined and tested. We demonstrate our technique to the solution of a classical boundary value problem in electrostatics, referring to a metallic and charged single-sheeted elliptic two-hyperboloid and its double-cone limit. Semi-analytical expressions for the related fields are derived, all cases being accompanied by the necessary numerical implementation.



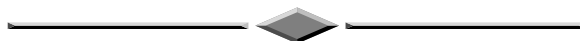
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48. P. Bakalis, P.K. Papadopoulos & P. Vafeas, “**Heat transfer study of the ferrofluid flow in a vertical annular cylindrical duct under the influence of a transverse magnetic field**”, *Fluids (Fluids)*, **6** (120), 1-11 (2021).
-

We study the laminar fully developed ferrofluid flow and heat transfer phenomena of an otherwise magnetic fluid into a vertical annular duct of circular cross-section and uniform temperatures on the walls, which is subjected to a transverse external magnetic field. A computational algorithm is used, which couples the continuity, momentum, energy, magnetization and Maxwell's equations, accompanied by the appropriate conditions, using the Continuity-Vorticity-Pressure (C.V.P.) method and a non-uniform grid. From the results, which are obtained for different values of the field strength and the volumetric concentration of the particles, it is evident that the ferrofluid flow and the temperature are affected by the magnetic field, wherein the effect on the axial velocity distribution is shown to be more intense. On the other hand, the dependence of the axial pressure gradient upon the field strength is almost linear, while the heat transfer significantly increases due to the generated secondary flow.



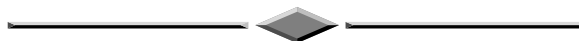
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49. P. Vafeas, P.K. Papadopoulos & P. Svarnas, “**Consideration of a mixed-type boundary value problem on the electrostatics of cold plasma jet reactors based on dielectric barrier discharge**”, *International Journal of Applied Mathematics and Computer Science (Int. J. Appl. Math. Comput. Sci.)*, **31**, 233-245 (2021).
-

A semi-analytical model is presented for the determination of the electric field in reactors used for Cold Atmospheric Pressure Plasma (CAPP) jet production, based on the Dielectric Barrier Discharge (DBD) concept. These systems are associated with diverse applications in contemporary engineering, ranging from material processing up to biomedicine, and at the same time they provide many challenges for fundamental research. Here, we consider a simplified system configuration of a single driven electrode, surrounding a thin dielectric tube, which does not contribute to the electric field, since the potential variation is immediate due to its negligible size. By employing the cylindrical coordinate system that perfectly fits the present plasma jet reactor, we separate the area of electric activity into three distinct domains according to the external imposed conditions, while our analysis is restricted to the electrostatic limit of Maxwell's equations. To this end, cylindrical harmonic field expansions are used for the potential, which produce the corresponding electric fields in each subdomain. Due to the imposed mixed-type boundary value problem, additional linear terms are incorporated, leading to three possible analytical solutions of the physical problem under consideration. The efficiency of the method is demonstrated by comparing the final formulae with a numerical solution, followed by the necessary discussion.



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50. M. Doschoris, A. Papargiri, V.S. Kalantonis & P. Vafeas, “**Application of boundary perturbations on medical monitoring and imaging techniques**”, *Nonlinear Analysis, Differential Equations, and Applications* (Springer: *Optimization and Its Applications*), **173**, 101-130 (2021).
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We present an overview on the application of boundary perturbations for Electroencephalography and Magnetoencephalography predominantly for the spherical geometry. With the mathematical tools produced, both forward and inverse problems can be tackled providing explicit computationally efficient solutions. Utilizing perturbation analysis in the framework of medical monitoring and imaging techniques, possesses the advantage introducing geometric variations without limiting the installation of analytic, or at least semi-analytic solutions, in view of complicated surfaces. In our example, surfaces which do not allow an analytic mathematical treatment can be handled if considered as small deviations from the sphere. In that setting, irregularities in head shapes, e.g. craniofacial alterations can be investigated theoretically.



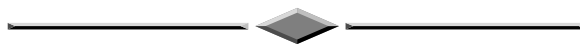
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51. A. Papargiri, V. Kalantonis, D. Kaziki, P. Vafeas & G. Fragoyiannis, “**Revisiting an analytical solution for the three-shell spherical human head model in electroencephalography**”, *Partial Differential Equations in Applied Mathematics (Partial Diff. Equ. Appl. Math.)*, **4 (100178)**, 1-6 (2021).
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The complete three-shell spherical human head model in electroencephalography (EEG) is revisited and an analytical solution of the forward problem is derived. The introduced geometrical model involves four concentric spheres that represent the successive interfaces between the cerebrum, the cerebrospinal fluid, the skull and the skin, which are characterized by different conductivities, while the outer environment is evidently the non-conductive air. The neuronal operation of the brain is considered to be represented by an equivalent and arbitrarily orientated electric dipole that is located in the inner sphere. The dipole source produces a bipolar primary current and the electric activity is initiated by means of the generated electric field, which is associated with the corresponding potential functions within each one of the conductive compartments of the model, inferring crucial information about EEG effects outside the head. The potentials formulae are obtained in compact fashion via the solution of a sequence of interconnected elliptic-type boundary value problems with Dirichlet and Neumann transmission conditions, where the consistent behavior of the fields in the brain and far away from the system has been taken into account. The efficiency of the suggested mathematical model is numerically implemented and the impact of the brain electric response to the exterior measurable potential field is demonstrated, implying a solid and sufficient head representation for the EEG forward problem.



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52. V. Papadimas, C. Doudehis, P. Svarnas, P.K. Papadopoulos, G.P. Vafakos & P. Vafeas, “**SDBD flexible plasma actuator with Ag-ink electrodes: Experimental assessment**”, *Applied Sciences (Appl. Sci.)*, **11 (11930)**, 1-13 (2021).
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In the present work, a single dielectric barrier discharge (SDBD) based actuator is developed and experimentally tested by means of various diagnostic techniques. Flexible dielectric barrier and conductive paint electrodes are used, making the designing concept applicable to surfaces of different aerodynamic profiles. The actuator technical drawing is given in detail. The plasma is sustained by audio frequency sinusoidal high voltage, while it is probed electrically and optically. The consumed electric power is measured, and the optical emission spectrum is recorded in the ultraviolet-near infrared (UV-NIR) range. High resolution spectroscopy provides molecular rotational distributions, which are treated appropriately to evaluate the gas temperature. The plasma induced flow field is spatiotemporally surveyed with pitot-like tube and schlieren imaging. Briefly, the actuator consumes a mean power less than 10 W, shows a fair stability over day scale period, the average temperature of the gas above its surface is close to 400 K and the fluid speed rises to 4.5 m s^{-1} . A long, thin layer (less than 1.5 mm) of laminar flow is unveiled on the actuator surface. This thin layer is interfaced with an outspread turbulent flow field, which occupies a cm-scale area. Molecular nitrogen positive ions appear to be part of the charged heavy species, in the generated filamentary discharge, which can transfer energy and momentum to the surrounding air molecules.



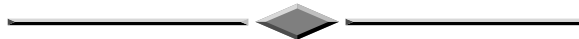
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53. E. Stefanidou, P. Vafeas & F. Kariotou, “**An analytical method of electromagnetic wave scattering by a highly conductive sphere in a lossless medium with low-frequency dipolar excitation**”, *Mathematics (Mathematics)*, **9** (3290), 1-25 (2021).
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The current research is involved with an analytical method of electromagnetic wave scattering by an impenetrable spherical object, which is immersed in an otherwise lossless environment. The highly conducting body is excited by an arbitrarily oriented time-harmonic magnetic dipole that is located at a reasonable remote distance from the sphere and operates at low frequencies for the physical situation under consideration, wherein the wavelength is much bigger than the size of the object. Upon this assumption, the scattering problem is formulated according to expansions of the implicated magnetic and electric fields in terms of positive integral powers of the wave number of the medium, which is linearly associated to the implied frequency. The static Rayleigh zeroth-order case and the initial three dynamic terms provide an excellent approximation for the obtained solution, while terms of higher orders are of minor significance and they are neglected, since we work at the low-frequency regime. To this end, the Maxwell's equations reduce to a finite set of interrelated elliptic partial differential equations, each one accompanied by the perfectly electrically conducting boundary conditions on the metal sphere and the necessary limiting behavior as we move towards the theoretical infinity, in practice very far from the observation domain. The presented analytical technique is based on the introduction of a suitable spherical coordinated system and yields compact fashioned three-dimensional solutions for the scattered components in view of infinite series expansions of spherical harmonic modes. In order to secure the validity and demonstrate the efficiency of this analytical approach, we invoke an example of reducing already known results from the literature to our complete isotropic case.



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54. D. Labropoulou, P. Vafeas & G. Dassios, “**Anisotropic elasticity and harmonic functions in Cartesian geometry**”, *Mathematical Analysis in Interdisciplinary Research* (Springer: *Optimization and Its Applications*), **179**, 523-553 (2021).
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Linear elasticity in an isotropic space is a well-developed area of Continuum Mechanics. However, the situation is exactly opposite if the fundamental space exhibits anisotropic behavior. In fact, the area of Linear Anisotropic Elasticity is not well-developed at the quantitative level, where actual closed form solutions are needed to be calculated. The present work aims to provide a little progress in this interesting branch of Continuum Mechanics. We provide a short review of Isotropic Elasticity in order to demonstrate in the sequel how the anisotropy modifies the final equations, via the Hooke's and Newton's laws. The eight standard anisotropic structures are also reviewed for completeness. A simple technique is introduced that generates homogeneous polynomial solutions of the anisotropic equations in Cartesian form. In order to demonstrate how this technique is applied, we work out the case of cubic anisotropy, which is the simplest anisotropic structure, having three independent elasticities. This choice is dictated by the restricted number of calculations it requires, but it carries all the basic steps of the method. Isotropic Elasticity accepts the differential representation of Papkovitch, which expresses the displacement field in terms of a vector and a scalar harmonic function. Unfortunately, though, no such representation is known for the anisotropic elasticity, which can represent the anisotropic displacement field in terms of solutions of the anisotropic Laplacian, as also discussed in this work.



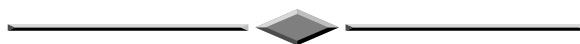
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55. P. Vafeas, E. Protopapas & M. Hadjinicolaou, “**On the analytical solution of the Kuwabara-type particle-in-cell model for the non-axisymmetric spheroidal Stokes flow via the Papkovich - Neuber representation**”, *Symmetry (Symmetry)*, **14** (170), 1-21 (2022).
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Modern engineering technology is involved with significant physical applications in heat and mass transfer, which are associated with the creeping motion of relatively homogeneous swarm of small particles, where the spheroidal geometry represents sufficiently the shape of the embedded particles within such aggregates. The steady Stokes flow of an incompressible, viscous fluid through an assemblage of particles, at low Reynolds numbers, is studied by employing a particle-in-cell model. The mathematical formulation adopts the Kuwabara-type assumption, according to which each spheroidal particle is stationary and it is surrounded by a confocal spheroid that creates a fluid envelope, in which the Newtonian fluid moves with a constant velocity of arbitrary orientation. The boundary value problem in the fluid envelope is solved by imposing non-slip conditions on the surface of the spheroid, which is considered also as non-penetrable, while zero vorticity is assumed on the fictitious spheroidal boundary along with a uniform approaching velocity. The three-dimensional flow fields are calculated for the first time analytically, in the spheroidal geometry, by virtue of the Papkovich - Neuber representation. Through this, the velocity and the total pressure fields are provided in terms of a vector and a scalar spheroidal harmonic potentials allowing this way the thorough study of the relevant physical characteristics of the flow fields. The newly obtained analytical expressions, generalize to any direction existing results holding for the asymmetrical case that were obtained with the aid of a stream function. These can be employed for the calculation of quantities of physical or engineering interest. Numerical implementation reveals the flow behavior within the fluid envelope for different geometrical cell characteristics and for arbitrarily assumed velocity field, reflecting this way the different flow / porous media situations. Sample calculations show an excellent agreement of the obtained results with the available for special geometrical cases ones. All these demonstrate the usefulness of the proposed method and the powerfulness of the obtained analytical expansions.



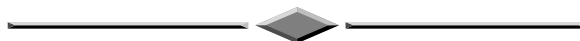
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56. G. Fragoyiannis, A. Papargiri, V.S. Kalantonis, M. Doschoris & P. Vafeas, “**Image reconstruction for positron emission tomography based on Chebyshev polynomials**”, *Approximation and Computation in Science and Engineering* (Springer: *Optimization and Its Applications*), **180**, 281-295 (2022).
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The study of the functional characteristics of the brain plays a crucial role in modern medical imaging. An important and effective nuclear medicine technique is Positron Emission Tomography (PET), whose utility is based upon the noninvasive measure of the in vivo distribution of imaging agents, which are labeled with emitting radionuclides. When dealing with applications that are associated with PET, we often face mathematical problems that involve the inverse Radon transform, leading to the development of several methods towards this direction. Herein, we present an improved formulation based on Chebyshev polynomials, according to which a novel numerical algorithm is employed in order to interpolate exact simulated values of the Radon transform via an analytical Shepp-Logan phantom representation. This approach appears to be efficient in calculating the Hilbert transform and its derivative, being incorporated within the final analytical formulae. The numerical tests are validated by comparing the presented methodology to the well-known spline reconstruction technique.



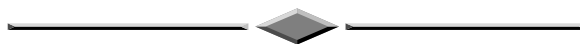
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57. G. Fragoyiannis, P. Vafeas & G. Dassios, “**On the reducibility of the ellipsoidal system**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **45**, 4497-4554 (2022).
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Lamé introduced a triaxial ellipsoidal system and via some ingenious arguments he managed to spectrally decompose the Laplace operator and define ellipsoidal harmonic functions. Since then, many authors modified the Lamé system and proposed some related ellipsoidal coordinate variables. Perhaps the most important of them is the system introduced by Jacobi, in connection with the geodesic curves on an ellipsoid. All other attempts to introduce ellipsoidal coordinates are basically modifications of the Jacobi or the Lamé system. However, an important question in the theory of ellipsoidal harmonics is the way in which this theory reduces to the spheroidal and spherical systems. This is by no means a straightforward procedure, since the relative limiting cases lead to generic underdetermined forms. The basic difficulty is due to the different dimensionality of the singularity regions corresponding to each system. Note that this region has zero dimensions in the case of the sphere, one dimension in the case of the prolate spheroid and two dimensions in the case of the oblate spheroid and the ellipsoid. The present work provides a systematic way to obtain these reductions and to establish a correspondence between ellipsoidal, spheroidal and spherical harmonics. We have also demonstrated how our approach applies to the proposed modified Jacobi ellipsoidal system when we connect the Lamé with the Jacobi system through certain non-degenerable relations. The importance of these geometrical degeneracies lies in the fact that the solution of any boundary value problem in ellipsoidal geometry provides immediately the solutions in all special geometries of prolate and oblate spheroids, discs, needles or spheres.



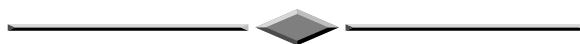
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58. A. Papargiri, V.S. Kalantonis, P. Vafeas, M. Doschoris, F. Kariotou & G. Fragoyiannis, “**On the geometrical perturbation of a three-shell spherical model in electroencephalography**”, *Mathematical Methods in the Applied Sciences* (*Math. Methods Appl. Sci.*), **45**, 8876-8889 (2022).
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The forward electroencephalography (EEG) problem is studied in the framework of a multilayered structure, which models the scalp, skull, cerebrospinal fluid and brain. Both the exterior and all inner boundaries are perturbed spheres so that special localized defects in head-brain imaging are considered in analytic fashion. Linear perturbation analysis is implemented, providing exact expression for the first significant term of the forward EEG solution of the perturbed problem. Comparison with the solution of the corresponding unperturbed spherical problem is included, together with numerical demonstration of the produced errors in special deformation cases. The results suggest that significant errors are caused when large inaccuracies in the head-brain structure in the vicinity of EEG source are not taken into account. Moreover, the suggested procedure provides a mathematical tool for evaluating quantitatively the impact of special deformations in the head representation on the EEG forward analytic solution.



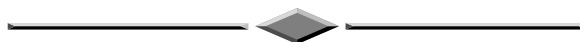
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59. D. Labropoulou, P. Vafeas & G. Dassios, “**Direct connection between Navier and spherical harmonic kernels in elasticity**”, *AIMS Mathematics* (*AIMS Math.*), **8**, 3064-3082 (2023).
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Linear isotropic elasticity is an interesting branch of continuum mechanics, described by the fundamental laws of Hooke and Newton, which are combined in order to construct the governing generalized Navier equation of the displacement within any material. Implying time-independence and in the absence of external body forces, the latter is reduced to the corresponding form of a homogeneous second-order partial differential equation, whose solution is given via the Papkovitch differential representation, which expresses the displacement field in terms of harmonic functions. On the other hand, spherical geometry provides the most widely used framework in real-life applications, concerning interior and exterior problems in elasticity. The present work aims to provide a little progress, by producing ready-to-use basic functions for linear isotropic elasticity in spherical coordinates. Hence, we calculate the Papkovitch eigen-solutions, generated by the spherical harmonic eigenfunctions, obtaining connections between Navier and spherical harmonic kernels. A set of useful results are provided at the end of the paper in the form of examples, regarding the evaluation of displacement field inside and outside a sphere.



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60. P.S. Stephanou, P. Vafeas & V.G. Mavrantzas, “**Non-equilibrium thermodynamics modelling of the stress-strain relationship in soft two-phase elastic-viscoelastic materials**”, *Journal of Non-Equilibrium Thermodynamics (J. Non-Equilib. Thermodyn.)*, **48**, 91-105 (2023).
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In “soft-soft nanocomposites” based on film formation of latexes with structured particles, the combination of particle structure and interparticle crosslinking leads to materials that behave as nonlinear viscoelastic fluids at small strains and as highly elastic networks at larger strains. Similarly, in studies of flow-induced crystallization in polymers, a two-phase model is often invoked, in which a soft viscoelastic component is coupled with a rigid semi-crystalline phase providing stiffness. In the present work, we use the framework of non-equilibrium thermodynamics (NET) to develop stress-strain relationships for such two-phase systems characterized by a viscoelastic and an elastic component by making use of two conformation tensors: the first describes the microstructure of the viscoelastic phase, while the second is related to the elastic Finger strain tensor quantifying the deformation of the elastic phase due to flow and is responsible for strain-hardening. The final transport equations are formulated in the context of the generalized bracket formalism of NET and can describe the rheological behavior and mechanical response of a large variety of soft materials, ranging from rubbers to artificial tissues.



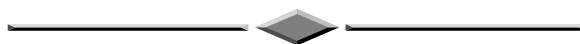
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61. D. Athanasopoulos, P. Svarnas, C. Liapis, P. Papadopoulos, K. Gazeli, K. Giotis, P. Vafeas, G. Vafakos, V. Giannakakis & A. Gerakis, “**Combination of ICCD fast imaging and image processing techniques to probe species-specific propagation due to guided ionization waves**”, *Physica Scripta (Phys. Scr.)*, **98** (055609), 1-19 (2023).
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The present report is devoted to the study of distinct ionization waves in terms of temporally-, spatially-, and wavelength-resolved analyses. The study is based on the technique of two-dimension fast imaging. However, appropriately selected ultraviolet to near infrared optical filters are employed to capture the propagation of specific species and digital image processing techniques are applied to explore the recorded snapshots. N_2 (SPS), N_2^+ (FNS), He I, OH(A-X), and O I, i.e., emissive neutral and ionic species, are investigated. On the other hand, the propagation of the NO_x species is studied by means of laser-induced fluorescence spectroscopy, since the light intensity due to spontaneous emission of this species was not readily detectable. Digital image processing techniques are also applied for the NO_x case. The crucial role of the above species in fields like plasma biomedicine and material processing is extensively recognized, and the present work: (i) provides gathered information on the propagation of these species within the atmospheric air; and (ii) introduces image processing algorithms to extract information which otherwise would remain hidden or uncorrelated with other plasma parameters. The present results unveil specific emission patterns due to the propagation of the N_2 (SPS), N_2^+ (FNS), He I, OH(A-X), and O I species. The intensity patterns consist of a first peak located in the vicinity of the reactor orifice, a second peak moving away from the reactor orifice, and a continuum which couples the two peaks. However, the detailed features of each pattern depend on the type of species considered. The fluorescence of the NO_x species also suggests the division of the area downstream of the reactor orifice into two regions where distinct ionization-excitation effects take place. Propagation speeds of the species up to about $2 \times 10^5 \text{ m s}^{-1}$ were measured. Finally, qualitative correlation between the species propagation (as it is pronounced by the emission patterns) and the local electric field (as it was numerically calculated in a similar setup) is demonstrated.



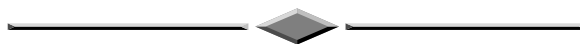
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62. P. Vafeas, A. Skarlatos, P.K. Papadopoulos, P. Svarnas & N. Sarmas, “**Boundary value problem of heat transfer within DBD-based plasma jet setups**”, *Mathematical Biosciences and Engineering (Math. Biosci. Eng.)*, **20**, 18345-18367 (2023).
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The present work claims an analytical solution for the thermal boundary value problem that arises in DBD-based plasma jet systems, as a preliminary and consistent approach to a simplified geometry. This approach involves the outline of a coaxial plasma jet reactor and the consideration of the heat transfer to the reactor solids, namely, the dielectric barrier and the grounded electrode. The non-homogeneous initial and boundary value thermal problem is solved analytically, while a simple cut-off technique is applied to deal with the appearance of infinite series relationships, being the outcome of merging dual expressions. The results are also implemented numerically, supporting the analytical solution, while a Finite Integration Technique (FIT) is used for the validation. Both the analytical and numerical data reveal the temperature pattern at the cross-section of the solids in perfect agreement. This analytical approach could be of importance for the optimization of plasma jet systems employed in tailored applications where temperature sensitive materials are involved, like in plasma biomedicine.



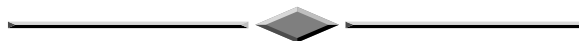
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63. G. Fragoyiannis & P. Vafeas, “**A semi-analytical approach for the computation of ellipsoidal harmonics**”, *Journal of Computational and Applied Mathematics (J. Comput. Appl. Math.)*, **437 (115418)**, 1-23 (2024).
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It is known that the separation of variables of the Laplace's equation in the ellipsoidal coordinate system leads to the Lamé equation, with solutions the Lamé functions of the first and second kind, which cannot be produced analytically for a degree higher than the seventh. These compose the corresponding ellipsoidal harmonic eigenfunctions, which are necessary for the complete description and solution of physical problems in three-dimensional domains with ellipsoidal boundaries. In this work, an efficient algorithm for calculating the Lamé functions of any degree and order is proposed, adopting compact and dimensionless expressions. The developed technique has been successfully applied to important applications in electrostatics and biophysics with excellent results. In addition, important complementary issues are analyzed, regarding the derivatives of the Lamé functions, the calculation of surface integrals, the Lamé functions of the second kind and at the same time we provide a closed-form expression for the normalization constants in terms of the polynomial coefficients of the Lamé functions.



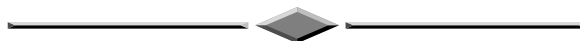
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64. P. Vafeas, “**A revisit of electromagnetic wave scattering by a metal isotropic body in a lossless environment with magnetic sensor excitation**”, *Sensors* (*Sensors*), **24** (3807), 1-17 (2024).
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This paper investigates the electromagnetic fields, being scattered by a metal spherical object in the vacuum environment, providing a numerical implementation of the obtained analytical results. A time-harmonic magnetic dipole source, far enough, emits at low frequencies the incident field, oriented arbitrarily in the three-dimensional space. The aim is to find a detailed solution to the scattering problem at spherical coordinates, useful for data inversion. Based on the theory of low frequencies, the Maxwell-type problem is transformed into Laplace's or Poisson's interconnected equations, accompanied by the proper boundary conditions on the perfectly conducting sphere and the radiation conditions at infinity, which are solved gradually. Approximating the static and the first three dynamic terms is sufficient, while the terms of higher order are negligible, which is confirmed by the field graphical representation.



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65. P. Bakalis, P. Papadopoulos & P. Vafeas, “**Heat transfer effect on the ferrofluid flow in a curved cylindrical annular duct under the influence of a magnetic field**”, *Computers & Mathematics with Applications (Comput. Math. Appl.)*, **170**, 172-185 (2024).
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The current research, which can be employed in various engineering applications, is involved with the investigation of the heat transfer effect on the laminar and fully developed ferrohydrodynamic flow into a curved annular cylindrical duct, when a constant very strong transverse magnetic field is applied. The numerical solution of the involved constitutive partial differential equations, i.e. the continuity, momentum, energy, magnetization and Maxwell's equations with the corresponding boundary conditions, is achieved via the computational Continuity-Vorticity-Pressure (C.V.P.) algorithmic method, using a conveniently chosen non-uniform grid. The method is implemented via an in-house code, which has been applied and validated in several ferrohydrodynamic flows. It incorporates a general theoretical model for the magnetohydrodynamic flow of micropolar magnetic fluids. The results show that the velocity distribution, the pressure drop and the temperature are significantly affected by the magnetic field strength and the volumetric concentration of the ferrofluid particles. The flow in the axial direction is redistributed in four symmetric poles, where its maximum value is readily observed. A secondary flow is generated, due to the combined effect of the buoyancy, the curvature and the magnetic field, which improves the heat transfer between the walls and the fluid. The axial pressure gradient, which is required to maintain the same mass flow, also increases as the field strength and concentration of magnetic particles increases, but with a lower rate in comparison to the increase in heat transfer.



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66. D. Labropoulou, T. Labropoulos, P. Vafeas & D.M. Manias, “**On the generalizations of the Cauchy-Schwarz-Bunyakovsky inequality with applications to elasticity**”, *Mathematical Analysis, Differential Equations and Applications* (World Scientific: *Computers and Operation Research*), **11 (18)**, 541-570 (2024).
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In this article we present both the discrete and the integral form of Cauchy- Bunyakovsky-Schwarz (CBS) inequality, some important generalizations in the n -dimensional Euclidean space and in linear subspaces of it, as well as the strengthened CBS. The last CBS inequality plays an important role in elasticity problems. A geometrical interpretation and a collection of the most important proofs of it are, also, presented.



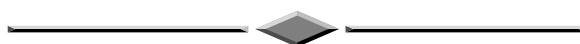
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67. P. Vafeas & P. Papadopoulos, “**A mathematical model for the nutrient distribution of a spheroidal avascular cancer tumour within an inhomogeneous environment**”, *Journal of Engineering Mathematics (J. Eng. Math.)*, **148 (2)**, 1-15 (2024).
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When a cancerous cell colony grows within a healthy environment, the entire structure can be modeled as a continuous two-phase fluid with five bounded compartments, governed by the laws of mass conservation, Fick’s diffusion law, and fluid mechanics principles. The interfaces of the five bounded compartments of the colony are defined by critical values of nutrient concentration. In studying the evolution of the exterior tumour boundary, nutrient concentration is the primary parameter. Although most existing research focuses on spherical tumours, significant implications for nutrient distribution emerge when spherical symmetry is abandoned, such as the occurrence of critical values at specific points rather than across the entire surface. In this work, we consider an oblate spheroidal tumour and investigate the effects of non-homogeneity in both nutrient supply and consumption rates. Our findings indicate that critical values are encountered within the interior of a thin layer, rather than at a single interface, although the interface is still included. We study the variation of nutrient concentration on the tumor’s interfaces through plots, highlighting the critical locations. The prolate spheroidal case can be derived via a simple transformation, while comparisons with similar spherical models are also discussed.



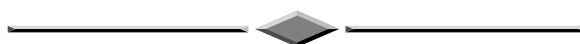
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68. P. Vafeas, “**Boundary value problems in ellipsoidal geometry and applications**”, *IgMin Research (IgMin Res.)*, **2**, 873-879 (2024).
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Many applications in science, engineering and modern technology require the solution of boundary value problems for genuine three-dimensional objects. These objects often are of, or can be approximated by, an ellipsoidal shape, where the three ellipsoidal semiaxes correspond to three independent degrees of freedom. The triaxial ellipsoid represents the sphere of any anisotropic space and for this reason it appears naturally in many scientific disciplines. Consequently, despite the complications of the ellipsoidal geometry and mainly its analysis, based on the theory of ellipsoidal harmonics, a lot of progress has been made in the solution of ellipsoidal boundary value problems, due to its general applicability. In this mini review, we aim to present to the scientific community the main achievements towards the investigation of three such physical problems of medical, engineering and technological significance, those comprising intense research in (a) electroencephalography (EEG) and magnetoencephalography (MEG), (b) creeping hydrodynamics (Stokes flow) and (c) identification of metallic impenetrable bodies, either embedded within the Earth’s conductive subsurface or located into a lossless air environment. In this context, special expertise and particular skills are needed in solving open boundary value problems that incorporate the ellipsoidal geometry and the related harmonic analysis, revealing the fact that there still exists the necessity of involving with these issues.



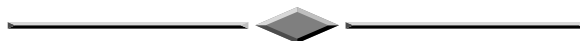
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69. E. Protopapas, P. Vafeas & M. Hadjinicolaou, “**A mathematical model for studying the Red Blood Cell magnetic susceptibility**”, *Applied Numerical Mathematics (Appl. Numer. Math.)*, **208**, 356-365 (2025).
-

The susceptibility of the human Red Blood Cells (RBCs) under the action of magnetic fields, either serves as a biomarker in medical tests, e.g. magnetic resonance imaging, nuclear magnetic resonance, magnetoencephalography or it is used in diagnostic and therapeutical processes, e.g. magnetophoresis for cell sorting. In the present manuscript, we provide analytical expressions for the magnetic potential and the magnetic field strength vector, when a magnetic field is applied to a RBC, modelled as a two-layered inverted spheroid. We introduce this way in the model the biconcave shape of the RBC and its structure (membrane and cytosol) in a more realistic representation, since, as until now, the RBC's shape was considered either as a sphere or a spheroid. The solution inside the RBC is obtained in R-separable form in terms of Legendre functions of the first and of the second kind and cyclic trigonometric functions, by applying appropriate boundary conditions on each layer. Our results reveal a non-uniform magnetic field inside the RBC. Parametric study of the solution, for various values of the physical properties of the RBC, is also provided, demonstrating the diamagnetic or the paramagnetic property of the RBC, which is strongly related to the health condition of the blood. The obtained solution may also serve for the justification of experimental results.



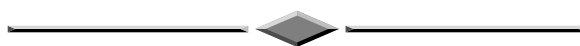
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70. D. Labropoulou, P. Vafeas, D.M. Manias & G. Dassios, “**Generalized solutions in isotropic and anisotropic elastostatics**”, *Journal of Elasticity (J. Elast.)*, **157** (34), 1-27 (2025).
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Linear elasticity comprises the fundamental branch of continuum mechanics that is extensively used in modern structural analysis and engineering design. In view of this concept, the displacement field provides a measure of how solid materials deform and become internally stressed due to prescribed loading conditions, a fact which is associated with linear relationships between the components of strain and stress, respectively. The mathematical characteristics of these dyadic fields are combined within the Hooke's law via the stiffness tetratic tensor, which embodies either the isotropic or the anisotropic behavior, exhibited by materials with linear properties. In fact, Hooke's law is incorporated into the general law of Newton that actually defines the principal spatial and temporal second-order non-homogeneous partial differential equation for the displacement. In this study, we construct handy closed-form solutions for Newton's law in the Cartesian regime, implying time-independence and considering the case of absence of body forces. Towards this direction, our aim is twofold, in the sense that an efficient analytical technique is introduced that generates homogeneous polynomial solutions of the displacement field for both the typical isotropic and the cubic-type anisotropic structure in the invariant Cartesian geometry. The reliability of the presented methodology is verified by reducing the results for each polynomial degree from the anisotropic to the isotropic eigenspace, in terms of a simple transformation, while we demonstrate our theory with an important application, wherein the effect of a prescribed force on an isotropic half-space to the neighboring half-space of cubic anisotropy is examined.



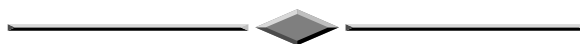
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71. V. Sevroglou, P. Vafeas & K.G. Lallas, “**Plane dyadic wave scattering by a small rigid body and cavity in 3D linear elasticity**”, *Mathematics (Mathematics)*, **13** (1975), 1-14 (2025).
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In this paper we study the 3D elastic scattering problem of plane dyadic waves for a rigid body and a cavity in linear elasticity. Initially, for each case we formulate the direct scattering problem in a dyadic form, and we give the corresponding longitudinal and transverse far-field scattering amplitudes. Due to dyadic formulation of the problems, the main outcome of this paper is to establish the necessary energy considerations as well as to present functionals and formulas for the differential and the scattering cross-section, in order to measure the disturbance created by the scatterer to the propagation of the plane dyadic incident field. Further, we assume that our incident field is scattered by a “small” rigid body or cavity and relative results for low-frequency scattering are obtained. Finally, we prove similar corresponding expressions for energy functionals in the far-field region along with expressions for the differential and the total scattering cross-section which are recovered as special cases.



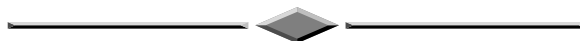
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72. G. Manoussakis & P. Vafeas, “**Spherical harmonics and gravity field modeling related to a special class of triaxial ellipsoids**”, *Mathematics (Mathematics)*, **13** (2115), 1-35 (2025).
-

The G-modified Helmholtz equation is a partial differential equation that allows gravity intensity g to be expressed as a series of spherical harmonics, with the radial distance r raised to irrational powers. In this study, we consider a non-rotating triaxial ellipsoid, parameterized by the geodetic latitude φ and geodetic longitude λ , and eccentricities e_e, e_x, e_y . On its surface, the value of gravity potential has a constant value, defining a level triaxial ellipsoid. In addition, the gravity intensity is known on the surface, which allows us to formulate a Dirichlet boundary value problem for determining the gravity intensity as a series of spherical harmonics. This expression for gravity intensity is presented here for the first time, filling a gap in the study of triaxial ellipsoids and spheroids. Given that the triaxial ellipsoid has very small eccentricities, a first order approximation can be made, by retaining only the terms containing $(e_e)^2$ and $(e_x)^2$. The resulting expression in spherical harmonics contains even degree and even order harmonic coefficients, along with the associated Legendre functions. The maximum degree and order that occurs is four. Finally, as a special case, we present the geometrical degeneration of an oblate spheroid.



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73. P. Vafeas, “**Electromagnetic excitation of a solid oblate spheroid in a lossless medium by a low-frequency magnetic dipole**”, *Journal of Engineering Mathematics (J. Eng. Math.)*, accepted (2025).
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The current contribution is associated with the electromagnetic wave scattering of an impenetrable oblate spheroidal object, which is embedded within a lossless medium (e.g. air) and it is illuminated by an arbitrarily orientated time-harmonic magnetic dipolar source that operates at considerably low frequencies, since it is assumed to be located in a far distance from the solid body. The physical situation admits a classical Maxwell-type boundary value problem and it is processed by expanding the involved magnetic and electric fields via positive integral powers of the wave number of the medium, which is linearly connected to the implied frequency. The Rayleigh static term of zeroth order and the three first dynamic terms of the series sufficiently describe the fields, because higher order terms are neglected due their minor contribution in the low-frequency regime. Evidently, the electromagnetic components of each remaining order constitute the fields under evaluation that comprise solutions of elliptic-type partial differential equations, which are accompanied by the appropriate boundary and limiting conditions. Implying the proper oblate spheroidal geometry, conveniently set at the center of the scatterer, the mathematical problem itself is readily solved, providing the spatial low-frequency electromagnetic fields in an analytical compact fashion as infinite series in terms of standard oblate spheroidal harmonic eigenfunctions and their counterparts. The validity of the presented analytical approach is demonstrated by including the reduction procedure for recovering the separate cases of a prolate spheroidal and a complete isotropic spherical target, hence a reliable benchmark is given.



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74. E. Protopapas, P. Vafeas & M. Hadjinicolaou, “**Stokes flow around a paraboloid of revolution: analytical treatment in an unbounded domain**”, *Physics of Fluids (Phys. Fluids)*, accepted (2025).
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Stokes flow describes the motion of a Newtonian, incompressible fluid in regimes where inertial effects are negligible compared to viscous forces. In the case of axisymmetric flow and by employing a stream function ψ the governing equation reduces to $E^4\psi = 0$, where E^2 is a second order elliptic partial differential operator and $E^4 = E^2 \circ E^2$. In this work, we derive the general solution of the equation $E^4\psi = 0$ in the parabolic coordinate system, which is given as series expansions of specific combinations of mixed order Bessel and modified Bessel functions of first and second kinds or as a polynomial. This analytical framework is applied to study the Stokes flow around a rigid paraboloid solving a boundary value problem, accordingly. A truncated error analysis is conducted, proving that using at least 8 terms of the obtained series expansion, the absolute error is less than 10^{-5} . Sample streamlines are depicted with respect to the order of truncation. The obtained stream function expansion allows for the computation of key hydrodynamic quantities, such as the velocity and the pressure fields, offering insights into the behavior of the particular flow. It may also serve as a foundation for further investigations of problems in a wide range of areas, from chemical engineering to biomedicine.

