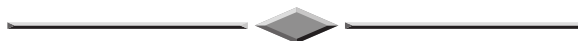


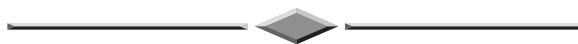
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1. G. Dassios & P. Vafeas, “**Connection formulae for differential representations in Stokes flow**”, *Journal of Computational and Applied Mathematics (J. Comput. Appl. Math.)*, **133**, 283–294 (2001).
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A pair of partial differential equations connecting the velocity with the total pressure field describes Stokes flow. Papkovich – Neuber and Boussinesq – Galerkin proposed two different differential representations of the flow fields (velocity and pressure) in terms of harmonic and biharmonic functions. On the other hand, spherical geometry provides the most widely used framework for representing small particles and obstacles embedded within a viscous, incompressible fluid characterizing the steady and non-axisymmetric Stokes flow. In the interest of producing ready-to-use basic functions for Stokes flow in spherical coordinates, we calculate the Papkovich – Neuber and the Boussinesq – Galerkin eigensolutions, generated by the well-known spherical harmonic and biharmonic eigenfunctions in the absence of singularities. Furthermore, connection formulae are obtained, by which we can transform any solution of the Stokes system from the Papkovich – Neuber to the Boussinesq – Galerkin eigenform and vice versa, only when the corresponding potentials have a particular form. We consider flows within internal and external domains with no singularity points on the axis of symmetry, where the two cases satisfy similar relations for the constant coefficients and their conjugates. Finally, we observe that the two general solutions are equivalent, but it is the differential representation Papkovich – Neuber that is given in a simpler form, as a result of the appearance of the biharmonic potential in the corresponding general solution proposed by Boussinesq – Galerkin.



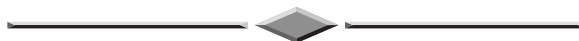
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2. P. Vafeas, “**On the connection between Stokes and Papkovich – Neuber spherical eigenfunctions in Stokes flow**”, *Bulletin of the Greek Mathematical Society (Bull. Greek Math. Soc.)*, **47**, 59–73 (2003).
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Stokes flow characterizes the steady and non-axisymmetric flow of an incompressible, viscous fluid at low Reynolds number and is described by a pair of partial differential equations connecting the velocity with the pressure field. Spherical geometry provides the most widely used framework for representing small particles embedded within a fluid that flows according to Stokes law and thus, the flow is assumed to be axisymmetric. The two different complete representations of the flow fields are considered here. The first one, named Stokes representation, is obtained, expressing the equation of motion in spherical coordinates, according to which the stream function is given in full series expansion in terms of separable eigenmodes. The second one, also valid in non-axisymmetric geometries, is the Papkovich – Neuber differential representation, where the flow fields are provided in terms of harmonic spherical eigenfunctions. In the interest of producing ready-to-use basic functions for axisymmetric Stokes flow in spherical coordinates by showing the different approach of solving such problems, we calculate the Stokes (2-D) and Papkovich – Neuber (3-D) eigen-solutions, demonstrating the full series expansion. In the present work, connection formulae are obtained which relate the spherical harmonic eigenfunctions of the Papkovich – Neuber representation, considering rotational symmetry, with the separable spherical stream eigenfunctions, excluding singularities. In that way, we transform any solution of the Stokes symmetric system from one representation to the other taking advantage of each one, as the case may be.



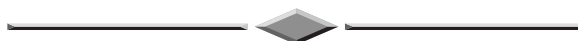
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3. G. Dassios & P. Vafeas, “**Comparison of differential representations for radially symmetric Stokes flow**”, *Abstract and Applied Analysis (Abstr. Appl. Anal.)*, **4**, 347–360 (2004).
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The steady and creeping flow for incompressible viscous fluids is described by the well-known Stokes equations, connecting the biharmonic velocity with the harmonic total pressure field. Palaniappan – Nigam – Amaranath – Usha (PNAU) and Papkovich – Neuber (PN) proposed two different representations of the velocity and the pressure in terms of harmonic and biharmonic functions, which form a practical tool for many important physical applications. One of them is the particle-in-cell model for Stokes flow through a swarm of particles, which exhibits great theoretical and practical interest. Most of the analytical models in this realm consider spherical particles. Therefore, for many interior and exterior flow problems involving small particles, spherical geometry provides a very good approximation. In the interest of producing ready-to-use basic functions for Stokes flow in spherical coordinates, we calculate the PNAU and the PN eigensolutions, generated by the appropriate vector spherical harmonic, biharmonic eigenfunctions and the full series (no singularities) expansion is being demonstrated. Furthermore, connection formulae are obtained by which we can transform any solution of the Stokes system from the PN to the PNAU eigenform. We show that this procedure is not invertible since these formulae interrelate each PNAU potential with a specific combination of PN eigenfunctions, a fact that reflects the flexibility of the second representation. Hence, the advantage of the PN representation as it compares to the PNAU solution is established. This is demonstrated by solving the problem of the flow in a fluid cell filling the space between two concentric spherical surfaces with Kuwabara-type boundary conditions.



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4. G. Dassios, A.C. Payatakes & P. Vafeas, “**Interrelation between Papkovich – Neuber and Stokes general solutions of the Stokes equations in spheroidal geometry**”, *Quarterly Journal of Mechanics and Applied Mathematics (Quart. J. Mech. Appl. Math.)*, **57**, 181–203 (2004).
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Many practical applications involve particles (inorganic, organic, biological) with non-spherical but still axisymmetric shapes. The present work is concerned with some interesting aspects of the theoretical analysis of Stokes flow in spheroidal domains. Two different complete representations of Stokes flow are considered here. The first one is obtained through the theory of generalized eigenfunctions, according to which the stream function is expanded in terms of separable and *semiseparable* eigenfunctions. The second one, valid in non-axisymmetric geometries as well, is the Papkovich – Neuber differential representation, where the velocity and pressure fields are expressed in terms of harmonic spheroidal eigenfunctions. Connection formulae are obtained for the case of axisymmetric flows, which relate the spheroidal harmonic eigenfunctions of the Papkovich – Neuber representation with the semiseparable spheroidal stream eigenfunctions. In the case of axisymmetric spheroidal flows the Papkovich – Neuber approach is equivalent to the Stokes stream function approach, but the 3-D representation offers certain important advantages. Particle-in-cell models for Stokes flow through a swarm of particles are of substantial practical interest, because they provide a relatively simple platform for the analytical or semianalytical solution of heat and mass transport problems. The early versions of these models were concerned with spherical particles. For this reason particle-in-cell models for spheroidal particles were developed more recently. The flexibility of the Papkovich – Neuber differential representation is demonstrated by solving the problem of the flow in a fluid cell filling the space between two confocal spheroidal surfaces with Kuwabara-type boundary conditions.



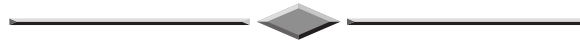
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5. P. Vafeas, G. Perrusson & D. Lesselier, “**Low–frequency solution for a perfectly conducting sphere in a conductive medium with dipolar excitation**”, *Progress in Electromagnetics Research (Prog. Electromagn. Res.)*, **49**, 87–111 (2004).
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This contribution concerns the interaction of an arbitrarily orientated, time–harmonic, magnetic dipole with a perfectly conducting sphere embedded in a homogeneous conductive medium. A rigorous low–frequency expansion of the electromagnetic field in positive integral powers $(ik)^n$, k complex wavenumber of the exterior medium, is constructed. The first $n=0$ vector coefficient (static or Rayleigh) of the magnetic field is already available, so emphasis is on the calculation of the next two nontrivial vector coefficients (at $n=2$ and at $n=3$) of the magnetic field. Those are found in closed form from exact solutions of coupled (at $n=2$, to the one at $n=0$) or uncoupled (at $n=3$) vector Laplace equations. They are given in compact fashion, as infinite series expansions of vector spherical harmonics with scalar coefficients (for $n=2$). The good accuracy of both in–phase (the real part) and quadrature (the imaginary part) vector components of the diffusive magnetic field is illustrated by numerical computations in a realistic case of mineral exploration of the Earth by inductive means. This canonical representation, not available yet in the literature to this time (beyond the static term), may apply to other practical cases than this one in geoelectromagnetics, whilst it adds useful reference results to the already ample library of scattering by simple shapes using analytical methods.



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6. A. Andrikopoulos & P. Vafeas, “**Maximal elements for binary relations on compact spaces**”, *Italian Journal of Pure and Applied Mathematics (Ital. J. Pure Appl. Math.)*, **19**, 85–90 (2006).
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The problem of characterizing the existence of maximal elements for preference relation has been stated firstly by Bergstrom in 1975. In this paper we give necessary and sufficient continuity conditions for the existence of maximal elements for a binary relation defined on a compact set.



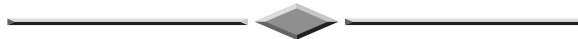
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7. G. Dassios & P. Vafeas, “**The 3D Happel model for complete isotropic Stokes flow**”, *International Journal of Mathematics and Mathematical Sciences (Int. J. Math. Math. Sci.)*, **46**, 2429–2441 (2004).
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Particle-in-cell models for the steady and non-axisymmetric flow of incompressible, viscous fluids at low Reynolds number (Stokes flow) are useful in the development of simple but reliable analytical expressions for swarms of particles. Most of the analytical models in this realm consider spherical particles. Therefore, spherical geometry provides the most widely used framework for representing small particles embedded within a fluid that flows according to Stokes description. Despite the fact that many physical applications involve particles with axisymmetric shapes leading to radially symmetric flows, it is of great theoretical and practical interest to investigate three-dimensional flow in assemblages of such particles. Here, the creeping flow through a swarm of spherical particles that move with constant velocity in an arbitrary direction and rotate with an arbitrary constant angular velocity in a quiescent Newtonian fluid is analyzed with a 3-D sphere-in-cell model. The mathematical treatment is based on the two concentric spheres model. The inner sphere comprises one of the particles in the swarm and the outer sphere consists of a fluid envelope. The appropriate boundary conditions of this non-axisymmetric formulation are similar to those of the 2-D sphere-in-cell Happel model, namely, non-slip flow condition on the surface of the solid sphere and nil normal velocity component and shear stress on the external spherical surface. The boundary value problem is solved with the aim of the complete Papkovitch – Neuber differential representation of the solutions for Stokes flow, which is valid in non-axisymmetric geometries and provides us with the velocity and total pressure fields in terms of harmonic spherical eigenfunctions. The solution of this three-dimensional model, which is self-sufficient in mechanical energy, is obtained in closed form and analytical expressions for the velocity, the total pressure, the angular velocity and the stress tensor fields are provided.



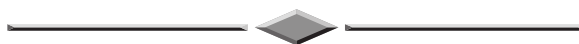
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8. P. Vafeas, “**Distribution of spheroidal focal singularities in Stokes flow**”, *International Journal of Pure and Applied Mathematics (Int. J. Pure Appl. Math.)*, **22**, 329–339 (2005).
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Stokes flow for the steady, non-axisymmetric motion of viscous, incompressible fluids in small Reynolds numbers (creeping flow), around small particles embedded within simply connected and bounded flow domains, is described by a pair of partial differential equations, which evolve the vector biharmonic velocity and the scalar harmonic total pressure fields. There exist many representations of the solutions of those kinds of flows, in three-dimensional domains, appearing in the form of differential operators acting on harmonic and biharmonic potentials. On the other hand, the development of Stokes theory for two-dimensional flows has the advantage that uses only one potential function (stream function) for the representation of the flow fields, but refers to axisymmetric flows. The effect of a distribution of sources – singularities, on the surface of a spheroidal particle or marginally on the focal segment, to the basic flow fields, is the goal of the present work. In particular, the proper confrontation of the problem is ensured by the introduction of the well-known Havelock’s theorem for the presence of singularities, which provides us with the necessary integral representations of the velocity and the pressure. Moreover, the interrelation of the eigenforms of the Papkovitch – Neuber differential representation with those that arise from Stokes theory, in two-dimensional spheroidal geometry, completes the two manners of facing the problem.



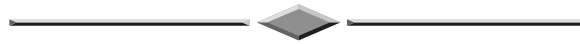
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9. P. Vafeas & G. Dassios, “**Stokes flow in ellipsoidal geometry**”, *Journal of Mathematical Physics (J. Math. Phys.)*, **47 (093102)**, 1–38 (2006).
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Particle-in-cell models for Stokes flow through a relatively homogeneous swarm of particles are of substantial practical interest, because they provide a relatively simple platform for the analytical or semianalytical solution of heat and mass transport problems. Despite the fact that many practical applications involve relatively small particles (inorganic, organic, biological) with axisymmetric shapes, the general consideration consists of rigid particles of arbitrary shape. The present work is concerned with some interesting aspects of the theoretical analysis of creeping flow in ellipsoidal, hence non-axisymmetric domains. More specifically, the low Reynolds number flow of a swarm of ellipsoidal particles in an otherwise quiescent Newtonian fluid, that move with constant uniform velocity in an arbitrary direction and rotate with an arbitrary constant angular velocity, is analyzed with an ellipsoid-in-cell model. The solid internal ellipsoid represents a particle of the swarm. The external ellipsoid contains the ellipsoidal particle and the amount of fluid required to match the fluid volume fraction of the swarm. The non-slip flow condition on the surface of the solid ellipsoid is supplemented by the boundary conditions on the external ellipsoidal surface which are similar to those of the sphere-in-cell model of Happel (self-sufficient in mechanical energy). This model requires zero normal velocity component and shear stress. The boundary value problem is solved with the aim of the potential representation theory. In particular, the Papkovitch – Neuber complete differential representation of Stokes flow, valid for non-axisymmetric geometries, is considered here, which provides the velocity and total pressure fields in terms of harmonic ellipsoidal eigenfunctions. The flexibility of the particular representation is demonstrated by imposing some conditions, which made the calculations possible. It turns out that the velocity of first degree, which represents the leading term of the series, is sufficient for most engineering applications, so long as the aspect ratios of the ellipsoids remains within moderate bounds. Analytical expressions for the leading terms of the velocity, the total pressure, the angular velocity and the stress tensor fields are obtained. Corresponding results for the prolate and the oblate spheroid, the needle and the disk, as well as for the sphere are recovered as degenerate cases. Novel relations concerning the ellipsoidal harmonics are included in appendix.



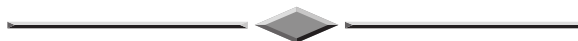
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10. V. Sevroglou & P. Vafeas, “**2D elastic scattering of a plane dyadic wave by a small rigid body and cavity**”, *ZAMM – Journal of Applied Mathematics and Mechanics (Z. Angew. Math. Mech.)*, **88**, 227–238 (2008).
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In this paper the scattering problem of the disturbance of a plane dyadic wave by a rigid body or a cavity, in two-dimensional elastodynamics is considered. The direct scattering problems are formulated in a dyadic form, and in each case, the corresponding longitudinal and transverse far-field scattering amplitudes are presented. We provide the necessary energy considerations and expressions for the differential and the scattering cross-section due to plane wave dyadic incidence. Next, the rigid body and cavity are considered to be small and are illuminated by a plane dyadic field. Finally, relative results for low-frequency scattering are obtained, and similar corresponding expressions for energy functionals in the far-field along with expressions for the differential and the total scattering cross-section are recovered as special cases.



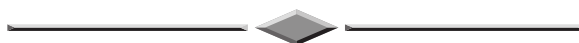
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11. G. Dassios & P. Vafeas, “**On the spheroidal semiseparation for Stokes flow**”, *Research Letters in Physics (Res. Lett. Phys.)*, **Volume 2008** (Article ID 135289), 1–4 (2008).
-

Many heat and mass transport problems involve particle–fluid systems, where the assumption of the Stokes flow consideration provides a very good approximation for representing small particles embedded within a viscous, incompressible fluid characterizing the steady, creeping flow. The present work is concerned with some interesting practical aspects of the theoretical analysis of Stokes flow in spheroidal domains. The stream function ψ , for axisymmetric Stokes flow, satisfies the well-known equation $E^4\psi = 0$. Despite the fact that in spherical coordinates this equation admits separable solutions, this property is not preserved when one seeks solutions in the spheroidal geometry. Nevertheless, defining some kind of semiseparability, the complete solution for ψ in spheroidal coordinates has been obtained in the form of products combining Gegenbauer functions of different degree. Thus, the general solution is represented in a full series expansion in terms of eigenfunctions, which are elements of the space $\ker E^2$ (separable solutions) and in terms of generalized eigenfunctions, which are elements of the space $\ker E^4$ (semiseparable solutions). In this work we revisit this aspect by introducing a different and simpler way of representing the aforementioned generalized eigenfunctions. Consequently, additional semiseparable solutions are provided in terms of the Gegenbauer functions, whereas the completeness is preserved and the full series expansion is rewritten in terms of these functions.



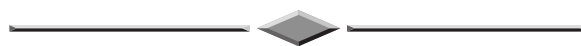
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12. P. Vafeas, G. Perrusson & D. Lesselier, “**Low-frequency scattering from perfectly conducting spheroidal bodies in a conductive medium with magnetic dipole excitation**”, *International Journal of Engineering Science (Int. J. Eng. Sci.)*, **47**, 372–390 (2009).
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Inductive electromagnetic means that are currently employed in the exploration of the Earth’s subsurface and embedded voluminous bodies often call for an intensive use, primary at the modeling stage and later on at the inversion stage, of analytically demanding tools of field calculation. Under the aim of modeling implementation, this contribution is concerned with some interesting aspects of the low-frequency interaction of arbitrarily orientated (i.e. three-dimensional) time-harmonic magnetic dipoles, with 3-D perfectly conducting spheroidal bodies embedded in an otherwise homogeneous conductive medium. For many practical applications involving buried obstacles such as Earth’s subsurface electromagnetic probing at low-frequency or any other physical cases (e.g. geoelectromagnetics), non-axisymmetric spheroidal geometry approximates sufficiently such kind of metallic shapes. On the other hand, our analytical approach deals with prolate spheroids, since the corresponding results for the oblate spheroidal geometry can be readily obtained through a simple transformation. The particular physical model concerns a solid impenetrable (metallic) body under a magnetic dipole excitation, where the scattering boundary value problem is attacked via rigorous low-frequency expansions for the incident, scattered and total electric and magnetic fields in terms of positive integral powers of (ik) , that is $(ik)^n$ for $n \geq 0$, where k stands for the complex wavenumber of the exterior medium. The purpose of the modeling is to contribute to a simple yet versatile tool to infer information on an unknown body from measurements of the three-component electric and magnetic fields nearby. Our goal is to obtain the most important terms of the low-frequency expansions of the electromagnetic fields, that is the static (for $n = 0$) and the dynamic ($n = 1, 2, 3$) terms. In particular, for $n = 1$ there are no incident fields and thus no scattered ones, while for $n = 0$ the Rayleigh electromagnetic expression is easily obtained in terms of infinite series. Emphasis is given on the calculation of the next two non-trivial terms (at $n = 2$ and at $n = 3$) of the aforementioned fields. Consequently, those are found in closed form from exact solutions of coupled (at $n = 2$, to the one at $n = 0$) or uncoupled (at $n = 3$) Laplace equations and they are given in compact fashion, as infinite series expansions for $n = 2$ or finite forms for $n = 3$. Nevertheless, the difficulty of the Poisson’s equation that has to be solved for $n = 2$ is presented, whereas our analytical approach demands the use of the well-known cut-off method in order to obtain an analytical closed solution. Finally, this research adds useful reference results to the already ample library of scattering by simple shapes using analytical methods.



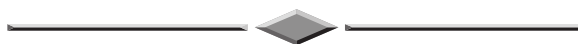
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13. P.M. Hatzikonstantinou & P. Vafeas “**A general theoretical model for the magnetohydrodynamic flow of micropolar magnetic fluids. Application to Stokes flow**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **33**, 233–248 (2010).
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Many practical applications, which have an inherent interest of physical and mathematical nature, involve the hydrodynamic flow in the presence of a magnetic field. Magnetic fluids comprise a novel class of engineering materials, where the coexistence of liquid and magnetic properties provides us with the opportunity to solve problems with high mathematical and technical complexity. Here, our purpose is to examine the micropolar magnetohydrodynamic flow of magnetic fluids by considering a colloidal suspension of ferromagnetic material (usually non-conductive) in a carrier magnetic liquid, which is in general electrically conductive. In this case, the ferromagnetic particles behave as rigid magnetic dipoles. Thus, the application of an external magnetic field, apart from the creation of an induced magnetic field of minor significance, will prevent the rotation of each particle, will increase the effective viscosity of the fluid and will cause the appearance of an additional magnetic pressure. Despite the fact that the general consideration consists of rigid particles of arbitrary shape, the assumption of spherical geometry is a very good approximation as a consequence of their small size. Our goal is to develop a general three-dimensional theoretical model that conforms to physical reality and at the same time permits the analytical investigation of the partial differential equations, which govern the micropolar hydrodynamic flow in such magnetic liquids. Furthermore, in the aim of establishing the consistency of our proposed model with the principles of both ferrohydrodynamics and magnetohydrodynamics, we take into account both magnetization and electrical conductivity of the fluid, respectively. Under this aspect, we perform an analytical treatment of these equations in order to obtain the three-dimensional effective viscosity and total pressure in terms of the velocity field, of the total (applied and induced) magnetic field and of the hydrodynamic and magnetic properties of the fluid, independently of the geometry of the flow. Moreover, we demonstrate the usefulness of our analytical approach by assuming a degenerate case of the aforementioned method, which is based on the reduction of the partial differential equations to a simpler shape that is similar to Stokes flow for the creeping motion of magnetic fluids. In view of this aim, we use the potential representation theory to construct a new complete and unique differential representation of magnetic Stokes flow, valid for non-axisymmetric geometries, which provides the velocity and total pressure fields in terms of easy-to-find potentials, via an analytical fashion.



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14. G. Perrusson, P. Vafeas & D. Lesselier, “**Low-frequency dipolar excitation of a perfect ellipsoidal conductor**”, *Quarterly of Applied Mathematics (Quart. Appl. Math.)*, **68**, 513–536 (2010).
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This paper deals with the scattering by a perfectly conductive ellipsoid under magnetic dipolar excitation at low frequency. The source and the ellipsoid are embedded in an infinite homogeneous conducting ground. The main idea is to obtain an analytical solution of this scattering problem in order to have a fast numerical estimation of the scattered field that can be useful for real data inversion. Maxwell equations and boundary conditions, describing the problem, are firstly expanded using low-frequency expansion of the fields up to order three. It will be shown that fields have to be found incrementally. The static one (term of order zero) satisfies the Laplace equation. The next non-zero term (term of order two) is more complicated and satisfies the Poisson equation. The order-three term is independent of the previous ones and is described by the Laplace equation. They constitute three different scattering problems that are solved using the separated variables method in the ellipsoidal coordinate system. Solutions are written as expansions on the few analytically known scalar ellipsoidal harmonics. Details are given to explain how those solutions are achieved with an example of numerical results.



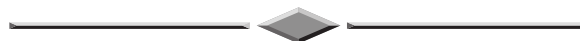
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15. P. Vafeas, P.K. Papadopoulos & P.M. Hatzikonstantinou, “**On the perturbation of the three-dimensional Stokes flow of micropolar fluids by a constant uniform magnetic field in a circular cylinder**”, *Mathematical Problems in Engineering (Math. Probl. Eng.)*, **Volume 2011** (Article ID 659691), 1–41 (2011).
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Several practical applications in engineering technology involve the micropolar magnetohydrodynamic flow of magnetic fluids in the presence of a magnetic field. Here, we consider a colloidal suspension of non-conductive ferromagnetic material, which consists of very small spherical particles that behave as rigid magnetic dipoles, in a carrier liquid of approximately zero conductivity and low-Reynolds number properties. The interaction of a 3-D constant uniform magnetic field with the three-dimensional steady creeping motion (Stokes flow) of a viscous incompressible micropolar fluid in a circular cylinder is investigated, where the magnetization of the ferrofluid has been taken into account. We use a degenerate case of the general three-dimensional theoretical model that governs the micropolar hydrodynamic flow in such liquids, which is based on the reduction of the partial differential equations to a simpler shape that is similar to Stokes flow. Those magnetic Stokes equations contain the additional effective viscosity of the fluid due to the ferromagnetic particles in terms of the applied magnetic field and of both the hydrodynamic and magnetic properties of the fluid, independently of the geometry of the flow. Our goal is to apply the proper boundary conditions that conform to physical reality, so as to obtain the flow fields in a closed analytical form via the potential representation theory, and to study several characteristics of the flow. In view of this aim, we make use of an improved new complete and unique differential representation of magnetic Stokes flow, valid for non-axisymmetric geometries, which provides analytically the velocity and total pressure fields in terms of easy-to-find potentials. We use these results to simulate the creeping flow of a magnetic fluid inside a circular duct and to obtain the flow fields associated with this kind of flow.



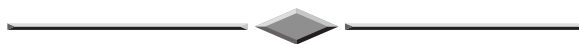
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16. P. Vafeas, P.K. Papadopoulos & D. Lesselier, “**Electromagnetic low–frequency dipolar excitation of two metal spheres in a conductive medium**”, *Journal of Applied Mathematics (J. Appl. Math.)*, **Volume 2012** (Article ID 628261), 1–37 (2012).
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This work concerns the low–frequency interaction of a time–harmonic magnetic dipole, arbitrarily orientated in the three–dimensional space, with two perfectly conducting spheres embedded within a homogeneous conductive medium. In several applications, where two bodies are placed near one another, the 3–D bispherical geometry provides a good approximation. The particular physical problem is modeled by considering two solid impenetrable (metallic) obstacles, excited by a magnetic dipole, where the scattering boundary value problem is attacked via rigorous low–frequency expansions in terms of integral powers $(ik)^n$, where $n \geq 0$, k being the complex wave number of the exterior medium, for the incident, scattered and total electric and magnetic fields. We deal with the most important terms of the low–frequency expansions of the non–axisymmetric scattered electromagnetic fields, that is the static (for $n = 0$) and the dynamic ($n = 1, 2, 3$) terms, while for $n \geq 4$ the contribution of the additional terms is of minor significance. The calculation of the exact solutions, satisfying Laplace’s and Poisson’s differential equations, leads to infinite linear systems, solved approximately within any order of accuracy through a cut–off procedure and via numerical implementation. Thus, we obtain the electromagnetic fields in an analytically compact fashion as infinite series expansions of bispherical eigenfunctions. This particular electromagnetic scattering problem is then simulated in order to investigate the effect of the radii ratio, the relative position of the spheres and the position of the dipole on the real and imaginary parts of the calculated scattered magnetic field.



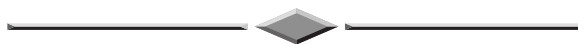
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17. F. Kariotou & P. Vafeas, “**The avascular tumour growth in the presence of inhomogeneous physical parameters imposed from a finite spherical nutritive environment**”, *International Journal of Differential Equations (Int. J. Differ. Equations)*, **Volume 2012** (Article ID 175434), 1–25 (2012).
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A well-known mathematical model of radially symmetric tumour growth is revisited in the present work. Under this aim a cancerous spherical mass lying in a finite concentric nutritive surrounding is considered. The host spherical shell provides the tumor with vital nutrients, receives the debris of the necrotic cancer cells and also transmits to the tumour the pressure imposed on its exterior boundary. We focus on studying the type of inhomogeneity that, the nutrient supply and the pressure field imposed on the host exterior boundary, can exhibit in order for the spherical structure to be supported. It turns out that if the imposed fields depart from being homogeneous, only a special type of interrelated inhomogeneity between nutrient and pressure can secure the spherical growth. The work includes an analytic derivation of the related boundary value problems based on physical conservation laws and their analytical treatment. Implementations in cases of special physical interest are examined and also existing homogeneous results from the literature are fully recovered.



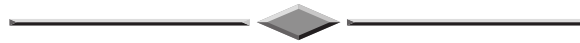
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18. P.K. Papadopoulos, P. Vafeas & P.M. Hatzikonstantinou, “**Ferrofluid pipe flow under the influence of the magnetic field of a cylindrical coil**”, *Physics of Fluids (Phys. Fluids)*, **24 (122002)**, 1–13 (2012).
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Ferrofluid pipe flow under the effect of a co-linear, finite length cylindrical coil is examined numerically. The specific flow configuration is chosen as it is encountered in engineering and bioengineering applications, such as magnetic drag targeting systems. The objective of the paper is twofold: Firstly, to investigate the accuracy of an analytical solution for the magnetization equation and assess its validity when used for non-uniform magnetic fields. It is found that it can be very helpful as a means of estimating the magnetization, especially for strong magnetic fields with low gradients. Secondly, to examine the effects of the magnetic field on the flow and study the relevant importance of the magnetic terms of the momentum equation. The parameters that we examine are the strength of the magnetic field and of its gradients, the volumetric concentration of the magnetic particles and the dimensions (length and diameter) of the coil. It is revealed that the axial pressure drop depends linearly on the volumetric concentration and that the magnetoviscosity effect is negligible in cases of non-uniform magnetic fields.



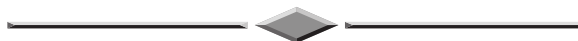
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19. G. Dassios, F. Kariotou & P. Vafeas, “**Invariant vector harmonics. The ellipsoidal case**”, *Journal of Mathematical Analysis and Applications (J. Math. Anal. Appl.)*, **405**, 652–660 (2013).
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We introduce a complete set of vector harmonic functions in an invariant form, that is, in a form that is independent of any coordinate system. In fact, we define three vector differential operators of the first order which, when they act on a scalar harmonic function they generate three independent vector harmonic functions. Then, we prove the relative independence properties and we investigate the characterization of every harmonic as irrotational or solenoidal field. We also prove that this set of functions forms a complete set of vector harmonics. Finally, we use these invariant expressions to recover the vector spherical harmonics of Hansen and to introduce vector ellipsoidal harmonics in \mathbb{R}^3 . Our method can be applied to any other coordinate system to produce the corresponding vector harmonics.



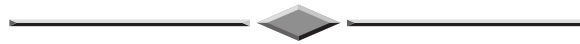
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20. K. Gazeli, P. Svarnas, P. Vafeas, P.K. Papadopoulos, A. Gkelios & F. Clément, “**Investigation on streamers propagating into a helium jet in air at atmospheric pressure: Electrical and optical emission analysis**”, *Journal of Applied Physics (J. Appl. Phys.)*, **114 (103304)**, 1–12 (2013).
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The plasma produced due to streamers guided by a dielectric tube and a helium jet in atmospheric air is herein studied electrically and optically. Helium streamers are produced inside the dielectric tube of a coaxial dielectric–barrier discharge and, upon exiting the tube, they propagate into the helium jet in air. The axisymmetric velocity field of the neutral helium gas while it penetrates the air is approximated with the PI-SO algorithm. At the present working conditions, turbulence helium flow is avoided. The system is driven by sinusoidal high voltage of variable amplitude (0–11 kV peak-to-peak) and frequency (5–20 kHz). It is clearly shown that a prerequisite for streamer development is a continuous flow of helium, independently of the sustainment or not of the dielectric–barrier discharge. A parametric study is carried out by scanning the range of the operating parameters of the system and the optimal operational window for the longest propagation path of the streamers in air is determined. For this optimum, the streamer current impulses and the spatiotemporal progress of the streamer UV–visible emission are recorded. The streamer mean propagation velocity is as well measured. The formation of copious reactive emissive species is then considered (in terms of intensity and rotational temperatures), and their evolution along the streamer propagation path is mapped. The main claims of the present work contribute to the better understanding of the physicochemical features of similar systems that are currently applied to various interdisciplinary engineering fields, including biomedicine and material processing.



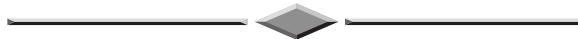
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21. F. Kariotou & P. Vafeas, “**On the transversally isotropic pressure effect on avascular tumor growth**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **37**, 277–282 (2014).
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A typical mathematical formulation for describing avascular tumour growth is considered in the present work, in the frame of oblate spheroidal geometry focusing on the kind of exterior conditions under which such a geometrical structure is attainable. It turns out that given a transversally isotropic pressure field, an avascular tumour can exhibit an oblate spheroidal growth, only if the nutrient supply is provided in a specific form related to the pressure and following the tumour evolution. A geometrical reduction to the prolate geometry and recovering of existing results for the sphere is included.



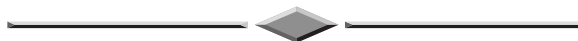
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22. F. Kariotou, P. Vafeas & P.K. Papadopoulos, “**Mathematical modeling of tumour growth in inhomogeneous spheroidal environment**”, *International Journal of Biology and Biomedical Engineering (Int. J. Biol. Biomed. Eng.)*, **8**, 132–141 (2014).
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Developing a mathematical model for cancer tumour growth that can be treated analytically and produce analytical results, is useful in the qualitative study of such complicated phenomenon. Most of such models consider radially symmetric tumours growing in homogeneous conditions, due to the availability of experimental data that concern mainly spherical tumours. Though, in vivo, the inhomogeneity of the host environment affects the geometrical features of the growing tumour mass, as shown in cases like the esophageal cancer. In the present work, we assume that the host tissue imposes the axisymmetric structure of a prolate spheroidal tumour via an appropriate pressure field and we investigate the evolution of such growth in a consistent nutritive microenvironment. To that purpose, the mathematical model that we consider consists of three boundary value problems, which describe the nutrient concentration, the inhibitor concentration and the pressure field in the interior and in the exterior of a layered prolate spheroid that models the tumour. These problems provide the necessary data for solving the evolution equation of the tumour’s exterior boundary, which is a highly nonlinear ordinary differential equation. Additionally, our model exhibits a geometrical reduction to special cases and, mainly, to the spherical geometry in order to recover the existing results for the sphere.



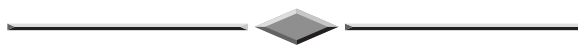
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23. P.K. Papadopoulos, P. Vafeas, P. Svarnas, K. Gazeli, P.M. Hatzikonstantinou, A. Gkelios & F. Clément, “**Interpretation of the gas flow field modification induced by guided streamer (‘plasma bullet’) propagation**”, *Journal of Physics D: Applied Physics (J. Phys. D: Appl. Phys.)*, **47 (425203)**, 1–16 (2014).
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Atmospheric–pressure non–equilibrium plasmas of noble gases in the form of “bullets” have attracted great attention, against cold low–pressure or thermal atmospheric–pressure plasmas, for multidisciplinary scientific fields such as material science and biomedicine, due to their unique compatible features. A key factor for the efficiency of most of these systems is the interaction between the noble–gas channel where the “bullets” (streamers) propagate and the plasma itself. It is the object of this article to demonstrate this interaction and to provide the explanation on the gas flow field modification induced by the plasma ignition. 3D numerical model incorporating most of the governing equations, schlieren imaging and UV–visible high resolution optical emission spectroscopy are applied. In accordance with the present results, the mechanism leading to the flow field alteration is clearly related to the electrohydrodynamic force, while it is demonstrated that the gas temperature plays a minor role.



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24. P. Svarnas, P.K. Papadopoulos, P. Vafeas, A. Gkelios, F. Clément & A. Mavon, “**Influence of atmospheric pressure guided streamers (plasma bullets) on the working gas pattern in air**”, *IEEE Transactions on Plasma Science (IEEE Trans. Plasma Sci.)*, **42**, 2430–2431 (2014).
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This work is devoted to the study of gas flow fields related to helium atmospheric pressure guided streamer (plasma bullet) propagation in the air. For very weak up to moderate helium flows, the modification induced to the gas flow field by the plasma ignition is demonstrated; it is shown that the turbulent flow region is expanded and two conditions must be fulfilled regarding the working gas profile in the air for streamer propagation, i.e., laminar flow and high concentration in this laminar flow region.



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25. M. Doschoris & P. Vafeas, “**Connection formulae between ellipsoidal and spherical harmonics with applications to fluid dynamics and electromagnetic scattering**”, *Advances in Mathematical Physics (Adv. Math. Phys.)*, **Volume 2015** (Article ID 572458), 1–12 (2015).
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The environment of the ellipsoidal system, significantly more complex than the spherical one, provides the necessary settings for tackling boundary value problems in anisotropic space. However, the theory of Lamé functions and ellipsoidal harmonics affiliated with the ellipsoidal system is rather complicated. A turning point would reside in the existence of expressions interlacing these two different systems. Still, there is no simple way, if at all, to bridge the gap. The present article addresses this issue. We provide explicit formulas of specific ellipsoidal harmonics expressed in terms of their counterparts in the classical spherical system. These expressions are then put into practice in the framework of physical applications.



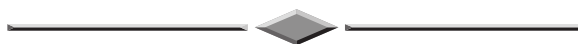
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26. P. Vafeas, P.K. Papadopoulos & P.M. Hatzikonstantinou, “**Analytical integro-differential representation of flow fields for the micropolar Stokes flow of a conducting ferrofluid**”, *IMA Journal of Applied Mathematics (IMA J. Appl. Math.)*, **80**, 839–864 (2015).
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Practical physical applications of mathematical nature are frequently met in engineering technology and involve the low-Reynolds number flow of micropolar conducting fluids under the effect of magnetic fields. Here, we consider the 3-D creeping motion (Stokes flow), in steady state, of a non-conductive colloidal suspension of ferromagnetic material embedded within an electrically conductive, viscous and incompressible, carrier liquid. In such cases the ferromagnetic particles behave as rigid magnetic dipoles and react in the presence of an externally applied magnetic field, which is of general form and arbitrarily orientated in the three-dimensional space. Therein, an induced magnetic field of minor significance is created, while the effective viscosity of the fluid is increasing and an additional magnetic pressure is appeared. The consistency of the governing set of partial differential equations with the principles of both ferrohydrodynamics and magnetohydrodynamics is established by taking into account magnetization and electrical conductivity of the fluid, respectively. Our main intension is to use the potential representation theory to improve previous models and construct a new complete and unique integro-differential representation of the magnetic Stokes flow of conducting liquids, valid for any non-axisymmetric geometry, which provides the velocity and total pressure fields in a closed form and in terms of easy-to-find potentials, via a semi-analytical formalism. In order to demonstrate the usefulness of our analytical approach, we assume a degenerate case of the aforementioned method to simulate the creeping flow of a micropolar fluid with conductive properties inside a circular duct.



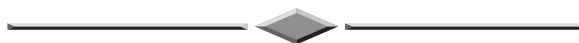
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27. G. Perrusson, P. Vafeas, I.K. Chatjigeorgiou & D. Lesselier, “**Low-frequency on-site identification of a highly conductive body buried in Earth from a model ellipsoid**”, *IMA Journal of Applied Mathematics (IMA J. Appl. Math.)*, **80**, 963–980 (2015).
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Identification of a highly-conductive orebody buried in Earth using an equivalent, perfectly-conducting, triaxial model ellipsoid is investigated. The real data available (three-component magnetic fields collected along a borehole due to a single-frequency current loop at the Earth surface) are simulated via a low-frequency, closed-form power series expansion of the electromagnetic fields scattered off an equivalent ellipsoid within a homogeneous, conductive medium, the source itself being idealized as a vertical magnetic dipole nearby. The approach provides formulations amenable to fast yet accurate computations, most of the work being in the construction of the formulations themselves, not in the numerical computations. The inversion scheme is described, which sees the iterative minimization of the least-square discrepancy between the fields due to a given ellipsoid and the data available. Unknowns are semi-axis lengths, angular orientations, and co-ordinates of its center. Numerical simulations illustrate the approach, before considering experimental single-well log data in a surface-to-borehole configuration at a mining site.



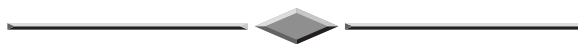
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28. P. Bakalis, P.M. Hatzikonstantinou & P. Vafeas, “**MFD formulations for the liquid metal flow in a curved pipe of circular cross section**”, *Computers & Fluids (Comput. Fluids)*, **119**, 1–12 (2015).
-

The laminar fully developed magnetohydrodynamic (MHD) flow of a liquid metal into a curved pipe of circular cross section, subjected to a transverse external magnetic field, is studied. Three different formulations are used for the implementation of the electromagnetic variables. The extended Continuity Vorticity Pressure (CVP) numerical variational method for MHD flows is used for the coupling of the momentum and the continuity equation. Results are obtained for different values of the curvature (0–0.2) and of the Hartmann number (0–1000). The magnitude of the axial velocity is determined by the balance of the centrifugal and the electromagnetic forces. The results reveal the limits of applicability of the used electromagnetic models as the Hartmann number increases.



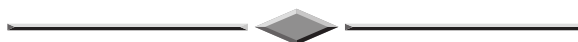
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29. P. Vafeas, D. Lesselier & F. Kariotou, “**Estimates for the low–frequency electromagnetic fields scattered by two adjacent metal spheres in a lossless medium**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **38**, 4210–4237 (2015).
-

Inductive electromagnetic means, currently employed in real physical applications and dealing with voluminous bodies embedded in lossless media, often call for analytically demanding tools of field calculation at modeling stage and later on at numerical stage. Here, one is considering two closely adjacent perfect conductors, possibly almost touching one another for which the 3–D bispherical geometry provides a good approximation. The particular scattering problem is modeled with respect to the two solid impenetrable metallic spheres, which are excited by a time–harmonic magnetic dipole, arbitrarily orientated in the three–dimensional space. The incident, the scattered and the total non–axisymmetric electromagnetic fields yield rigorous low–frequency expansions in terms of positive integral powers of the real–valued wave number in the exterior medium. We keep the most significant terms of the low–frequency regime, i.e. the static Rayleigh approximation and the first three dynamic terms, while the additional terms are small contributors and they are neglected. The typical Maxwell–type problem is transformed into intertwined either Laplace’s or Poisson’s potential–type boundary value problems with impenetrable boundary conditions. In particular, the fields are represented via 3–D infinite series expansions in terms of bispherical eigenfunctions, obtaining analytical closed–form solutions in a compact fashion. This procedure leads to infinite linear systems, which can be solved approximately within any order of accuracy through a cut–off technique.



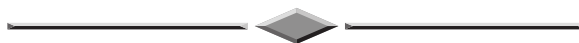
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30. M. Doschoris, G. Dassios, P. Vafeas, F. Kariotou & I.K. Chatjigeorgiou, “**Revisiting a numerical implementation of the EEG problem in ellipsoidal geometry**”, *Pioneer Journal of Advances in Applied Mathematics (Pioneer. J. Adv. Appl. Math.)*, **14**, 35–51 (2015).
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A triaxial ellipsoid provides an approximation of the average human brain which is much better than the broadly used spherical model. The analytical solution of the forward problem of the Electroencephalography (EEG) with an isolated dipolar source has been already derived and reported in the literature. This solution is expressed in terms of an eigenexpansion in ellipsoidal harmonics. Nevertheless, this expression was not possible to be handled effectively since no ellipsoidal harmonics of degree higher than seven were available in closed forms. In order to analyze further this problem an effective numerical algorithm has been developed, which generates the ellipsoidal harmonics of arbitrary degree and order in a numerical form. The algorithm has been compared with the known analytical eigenfunctions and the results manifested a perfect coincidence. Finally, this algorithm was used to construct a numerically stable solution of the electric potential on the surface of the head, which is generated by a single dipole of arbitrary position and orientation. The degree of ellipsoidal harmonics needed for a numerically convergent solution goes all the way up to 30 and the result provides a slightly improved model for tackling boundary value problems in ellipsoidal geometry.



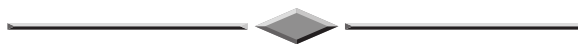
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31. P. Vafeas, P.K. Papadopoulos, P.–P. Ding & D. Lesselier, “**Mathematical and numerical analysis of low–frequency scattering from a PEC ring torus in a conductive medium**”, *Applied Mathematical Modelling (Appl. Math. Model.)*, **40**, 6477–6500 (2016).
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The electric and magnetic fields scattered off a non–penetrable ring torus, being characterized as perfect conductor, embedded in a homogeneous conductive medium and illuminated by a low–frequency magnetic dipole of arbitrary orientation and harmonic time–dependence are investigated herein. Upon definition of the complex wave number of the exterior medium k via its skin–depth, the 3–D scattering boundary value problem is handled via convenient low–frequency expansions in terms of powers of $(ik)^n$, $n \geq 0$ for the fields. A Maxwell–type problem is transformed into intertwined Laplace’s or Poisson’s potential–type boundary value problems with impenetrable boundary conditions. Using a toroidal coordinate system attached to the torus, they are solved as infinite series expansions for the fields in terms of toroidal eigenfunctions. In practice, what is accessible to the measurement is the scattered magnetic field. The static term ($n = 0$) provides most of its real (or in–phase) part and the second–order term ($n = 2$) consists of most of its imaginary part (quadrature), where in both cases a small contribution of the third–order term ($n = 3$) is being calculated. For $n = 1$, there exists no field, while the terms for $n \geq 4$ and for such kind of applications, have been proved to be of minor significance, hence they are neglected. The resulting infinite linear systems can be solved at any accuracy level through a cut–off process or via an analytical technique based on the method of finite continuous fraction solutions. Basics of the far–field approximation and the magnetic polarisability tensor are also included. At implementation stage, simulations are proposed in various situations, where a full–wave, finite–element approach is discussed.



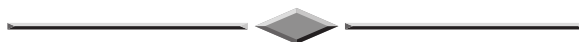
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32. P. Vafeas, “**Low–frequency electromagnetic scattering by a metal torus in a lossless medium with magnetic dipolar illumination**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **39**, 4268–4292 (2016).
-

The present contribution is concerned with an analytical presentation of the low–frequency electromagnetic fields, which are scattered off a highly conductive ring torus that is embedded within an otherwise lossless ambient and interacting with a time–harmonic magnetic dipole of arbitrary orientation, located nearby in the three–dimensional space. Therein, the particular 3–D scattering boundary value problem is modeled with respect to the solid impenetrable torus–shaped body, where the toroidal geometry fits perfectly. The incident, the scattered and the total non–axisymmetric magnetic and electric fields are expanded in terms of positive integral powers of the real–valued wave number of the exterior medium at the low–frequency regime, whereas the static Rayleigh approximation and the first three dynamic terms provide the most significant part of the solution, since all the additional terms are small contributors and, hence, they are neglected. Consequently, the typical Maxwell–type physical problem is transformed into intertwined either Laplace’s or Poisson’s potential–type boundary value problems with the proper conditions, attached to the metallic surface of the torus. The fields of interest assume representations via infinite series expansions in terms of standard toroidal eigenfunctions, obtaining in that way analytical closed–form solutions in a compact fashion. Although this mathematical procedure leads to infinite linear systems for every single case, these can be readily and approximately solved at a certain level of desired accuracy through standard cut–off techniques.



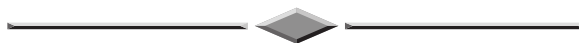
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33. D.K. Logothetis, P.K. Papadopoulos, P. Svarnas & P. Vafeas, “**Numerical simulation of the interaction between helium jet flow and an atmospheric–pressure “plasma jet”**”, *Computers & Fluids (Comput. Fluids)*, **140**, 11–18 (2016).
-

In this work we study the interaction of an atmospheric–pressure “plasma jet” with the hydrodynamic flow of the working gas. The study is based on the comparison between numerical simulation results and experimental data collected from the literature. Plasma reactors of three different configurations are considered, using a simple model, which focuses on the electro–hydrodynamic force importance. The objective is to evaluate the ability of the model to capture the resulting interaction between the “plasma jet” and the working gas for different reactor configurations. It is also aimed to find out possible correlations between the main parameters of the system, which may be useful for theoretical model development and reactor improved designing. In the context of the present model, it is assumed that the local electro–hydrodynamic force can be expressed via the product of a constant–motive part, which depends on the plasma setup and parameters, with the working gas local concentration, which expresses the dependence of the ionization rate on the gas concentration. The simulation results unveil that the constant–motive part is independent of the flow rate and inversely proportional to the diameter of the dielectric tube of the plasma reactor.



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34. J. Sten, G. Fragoyiannis, P. Vafeas, P. Koivisto & G. Dassios, “**Theoretical development of elliptic cross-sectional hyperboloidal harmonics and their application to electrostatics**”, *Journal of Mathematical Physics (J. Math. Phys.)*, **58 (053505)**, 1–19 (2017).
-

The analytic computation of electric and magnetic fields near corners and edges is important in many applications related to science and engineering. However, such complicated situations are hard to deal with, since they accumulate charges and consequently they mathematically represent singularities. In order to model this singular behavior, we introduce a novel method, which is related to the geometry and the analysis of the ellipsoidal coordinate system. Indeed, adopting the benefits of the corresponding coordinate surfaces, we use a general non-circular double cone, being the asymptote of a two-sided hyperboloid of two sheets with elliptic cross-section, which matches almost perfectly the particular physics and captures the corresponding essential features in a fully three-dimensional fashion. To this end, our analytical technique employs the ellipsoidal geometry and adapts the ellipsoidal functions (solutions of the well-known Lamé equation) so as to construct a new set of the so-called elliptic cross-sectional hyperboloidal harmonics, supplemented by the appropriate orthogonality rules on every constant coordinate surface. By first recollecting the key results of the coordinate system and the related potential functions, including the indispensable orthogonality results, we demonstrate our method to the solution of two boundary value problems in electrostatics. Both refer to a non-penetrable two-hyperboloid of elliptic cross-section and its double-cone limit, the first one being charged and the second one scattering off a plane wave. Closed form expressions are derived for the related fields, while the already known formulae from the literature are readily recovered, all cases being followed by the appropriate numerical implementation.



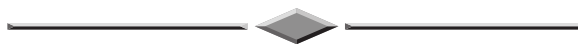
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35. P. Vafeas, “**Revisiting the low–frequency dipolar perturbation by an impenetrable ellipsoid in a conductive surrounding**”, *Mathematical Problems in Engineering* (*Math. Probl. Eng.*), **Volume 2017** (Article ID 9420658), 1–16 (2017).
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Real physical applications concerning underground detections or other identifications of voluminous bodies require analytically demanding tools of field calculation at modeling and numerical stage. This contribution deals with the scattering by a metallic ellipsoidal target, embedded in a homogeneous conductive medium, which is stimulated when a 3–D time–harmonic magnetic dipole is operating nearby at low frequency. The incident, the scattered and the total three–dimensional electromagnetic fields, which satisfy Maxwell’s equations, yield low–frequency expansions in terms of positive integral powers of the complex–valued wave number of the exterior medium. We preserve the most significant terms of the low–frequency realm, those being the static Rayleigh approximation and the first three dynamic terms, while the additional terms are neglected, since their contribution is minor. The Maxwell–type problem is transformed into intertwined either Laplace’s or Poisson’s potential–type boundary value problems with impenetrable boundary conditions. On the other hand, the environment of a genuine ellipsoidal coordinate system provides the necessary setting for tackling such problems in anisotropic space. The fields are represented via non–axisymmetric infinite series expansions in terms of harmonic eigenfunctions, affiliated with the ellipsoidal system, obtaining analytical closed–form solutions in a compact fashion. Until nowadays, such problems were attacked by using the very few ellipsoidal harmonics exhibiting an analytical form. In the present article, we address this issue by incorporating the full series expansion of the potentials and utilizing the entire subspace of ellipsoidal harmonic eigenfunctions. That way, it is feasible to introduce any numerical technique for implementing the evaluated fields up to very high orders of harmonics, depending on the desired accuracy for the convergence of the potential series.



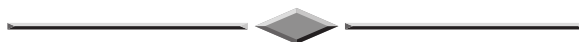
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36. P. Vafeas, “**On the integro–differential general solution for the unsteady micropolar Stokes flow of a conducting ferrofluid**”, *Quarterly of Applied Mathematics (Quart. Appl. Math.)*, **76**, 19–37 (2018).
-

The three–dimensional (3–D) unsteady creeping motion, corresponding to Stokes flow, of a non–conductive colloidal suspension of ferromagnetic particles, which are embedded within an otherwise electrically conducting, viscous and incompressible, carrier liquid, is considered in this contribution. This group of micropolar conducting ferrofluids comprises a novel class of engineering materials that respond in the presence of a general externally applied magnetic field, which is arbitrarily orientated in the three–dimensional domain of practical interest. Therein, an induced magnetic field of minor importance is created, while the effective viscosity of the fluid is increasing and an additional magnetic pressure is appeared. In order to be compatible with the principles of both ferrohydrodynamics and magnetohydrodynamics, we readily include the magnetization and the electrical conductivity of the magnetic fluid, respectively into the governing partial differential equations of the particular physical system. Employing the potential representation theory, we fabricate a new integro–differential general solution for the situation under investigation, which provides the time–dependent velocity and total pressure fields in a 3–D spaced closed form and in terms of easy–to–find potentials, via a semi–analytical shape. This generalized representation is proved to be complete, whilst it is valid for any non–axisymmetric geometry. We demonstrate the applicability of our analytical approach, by introducing a basic degenerate case of the aforementioned method to simulate the time–dependent creeping flow of a micropolar fluid with conductive properties inside a circular duct.



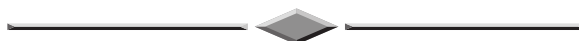
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37. M. Doschoris, P. Vafeas & G. Fragoyiannis “**The influence of surface deformations on the forward magnetoencephalographic problem**”, *SIAM Journal on Applied Mathematics (SIAM J. Appl. Math.)*, **78**, 963–976 (2018).
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A perturbational model is developed providing explicit computationally efficient solutions for the forward magnetoencephalographic problem, namely calculating the external magnetic fields for known neuronal sources. The aim of the study is to investigate the sensitivity of the particular measurements to deformations occurring on the conductor's surface. These geometric variations represent irregularities in head shapes and correspond to two major situations: (1) Localized acquired injuries of the scalp–skull delivered by external forces; (2) Craniofacial alterations due to natural mechanisms or defects. The presented methodology has the following advantages. Firstly, it supports the installation of tailored functions, which individually describe aforesaid deformations. Secondly, it allows rapid calculation of the forward problem for superficial cerebral activity, where similar numerical methods produce large errors. Our results indicate that surface deformations can have an eminent impact on magnetoencephalographic measurements under the condition that the neuronal brain activity is located beneath the deformed area, as well as on the extent of the deformation itself. In situations where surface deformations are not taken into account, the error made, varying between 5 to 25 per cent, propagates with distance.



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38. P. Vafeas, “**Dipolar excitation of a perfectly electrically conducting spheroid in a lossless medium at the low-frequency regime**”, *Advances in Mathematical Physics (Adv. Math. Phys.)*, **Volume 2018** (Article ID 9587972), 1–20 (2018).
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The electromagnetic vector fields, which are scattered off a highly conductive spheroid that is embedded within an otherwise lossless medium, are investigated in this contribution. A time-harmonic magnetic dipolar source, located nearby and operating at low frequencies, serves as the excitation primary field, being arbitrarily orientated in the three-dimensional space. The main idea is to obtain an analytical solution of this scattering problem, using the appropriate system of spheroidal coordinates, such as a possibly fast numerical estimation of the scattered fields could be useful for real data inversion. To this end, incident and scattered, as well total fields are written in a rigorous low-frequency manner in terms of positive integral powers of the real-valued wave number of the exterior environment. Then, the Maxwell-type problem is converted to interconnected either Laplace's or Poisson's equations, complemented by the perfectly conducting boundary conditions on the spheroidal object and the necessary radiation behavior at infinity. The static approximation and the three first dynamic contributors are sufficient for the present study, while terms of higher orders are neglected at the low-frequency regime. Henceforth, the 3-D scattering boundary value problems are solved incrementally, whereas the determination of the unknown constant coefficients leads either to concrete expressions or to infinite linear algebraic systems, which can be readily solved by implementing standard cut-off techniques. The non-axisymmetric scattered magnetic and electric fields follow and they are obtained in an analytical compact fashion via infinite series expansions in spheroidal eigenfunctions. In order to demonstrate the efficiency of our analytical approach, the results are degenerated so as to recover the spherical case, which validates this approach.



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39. P. Svarnas, P.K. Papadopoulos, D. Athanasopoulos, K. Sklias, K. Gazeli & P. Vafeas, “**Parametric study of thermal effects in a capillary dielectric–barrier discharge related to plasma jet production: Experiments and numerical modelling**”, *Journal of Applied Physics (J. Appl. Phys.)*, **124 (064902)**, 1–13 (2018).
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In the present work, a capillary dielectric–barrier discharge of coaxial electrode configuration, commonly employed to atmospheric–pressure cold plasma jet production, is studied in terms of thermal effects. The discharge is driven by sinusoidal high voltage in the kHz range and operates with helium gas channeled into a capillary dielectric tube having one end opened to the atmospheric air. The voltage amplitude and frequency, gas flow rate and discharge volume are varied independently, and thermal effects are investigated by experimentally acquired results coupled with numerically determined data. The experiments refer to electrical power and time–resolved temperature measurements and high resolution optical emission spectroscopy. The numerical modeling incorporates an electro–hydrodynamic force model in the governing equations to take into account the helium–air interplay and uses conjugate heat transfer analysis. The comparison between experimental and numerical data shows that power is principally consumed on the dielectric barrier and the gas phase reactions. A linear relation between steady state temperatures and supplied power, independently of the designing and operating conditions, is experimentally established. However, gas flow rate affects differently the thermal effects compared to the other parameters, supporting the idea of a two–fold nature of these systems, i.e. electrical and hydrodynamic. The main claim states the possibility of correlating designing and operating parameters for evaluating heat distribution and gas temperature trends in capillary dielectric–barrier discharges used for plasma jet production. This is of high importance for processing temperature–sensitive materials, including bio–specimens.



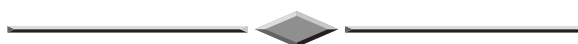
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40. P. Vafeas, A. Skarlatos, T. Theodoulidis & D. Lesselier, “**Semi-analytical method for the identification of inclusions by air-cored coil interaction in ferromagnetic media**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **41**, 6422–6442 (2018).
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The magnetostatic harmonic fields scattered by a near-surface air inclusion of arbitrary shape, embedded in a conductive ferromagnetic medium and illuminated by a current-carrying coil, are investigated. The scattering domain is separated into homogeneous subdomains under the assumption of a suitable truncation at a long distance from the incident source, whereas a perfect magnetic boundary condition is implied. The introduced methodology addresses the full coupling between the two interfaces, i.e. the plane that distinguishes the half-space ferromagnetic material from the open air and the arbitrary surface among the inclusion and the ferromagnetic region. Therein, continuity conditions are applied in a rigorous way, while the expected behavior of the fields, either as ascending or as descending, are taken into account. The potentials associated with the half-space are expanded via cylindrical harmonic eigenfunctions, while those related with the inclusion's arbitrary geometry admit generalized-type formalism. However, since the transmission conditions involve potentials with different eigenexpansions, we are obliged to rewrite cylindrical to generalized functions and vice versa, obtaining handy relationships in terms of easy-to-handle integrals, where orthogonality then is feasible. Once done, the calculation of the exact solutions leads to infinite linear algebraic systems, manipulated through standard cut-off techniques. Thus, we obtain the implicated fields in a general analytical and compact fashion, independent of the inclusion's geometry. We demonstrate the efficiency of the analytical model approach, assuming the degenerate case of a spherical inclusion, whereas the air-cored coil simulation via a numerical procedure validates our method. The calculation is very fast, rendering it suitable for use with parametric inversion algorithms.



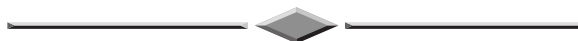
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41. G. Gavriil, P. Vafeas, A. Kanavouras & F.A. Coutelieras, “**Validation method for the systematization of results based on a similarity concept**”, *Mathematical Methods in the Applied Sciences (Math. Methods Appl. Sci.)*, **42**, 656–666 (2019).
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The development of a functional methodological approach is presented here, to clarify a globally valid way of evaluating the precision of mathematical modeling of physical and/or chemical processes. Starting from the description of the system, a phenomenon accompanied by a disclaiming hypothesis is investigated, against which the knowledge is accumulated with time. Moreover, the possibility of the evolution of any phenomenon being interrupted when a parameter overpasses a critical threshold, after which the hypothesis is not any more valid, is introduced. This possibility should be obtained through the dependence of a selected macroscopic quantity (marker) on a specific parameter. To apply this methodology, the problem of Stokes flow through a granular medium of spheroidal grains has been selected as an indicative case study. The prolate spheroidal configuration is considered, since the results for the oblate spheroid can be recovered via simple transformation. Therein, the three-dimensional flow fields are initially constructed analytically via the Papkovich – Neuber differential representation, which provides the velocity and pressure fields in terms of harmonic spheroidal eigenfunctions. Next, under the Kuwabara-type spheroidal 2D unit cell concept, the above expressions degenerate to the axisymmetric case and the full solution is then obtained, keeping the leading terms of the series, which are adequate for most engineering applications for specific aspect ratio of the spheroids. In the sequel, the aforementioned problem is solved numerically for a 3D extension of the same model, where this numerical solution has been achieved by using the Finite Volumes Method (FVM), while the resulting linear systems were approximated by applying the well-known Successful Over-Relaxation (SOR) concept. Finally, outcomes by both models have been compared via the above methodology, resulting to objective and reliable accuracy criteria.



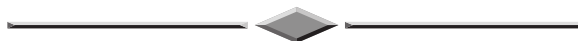
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42. P. Papadopoulos, D. Athanasopoulos, K. Sklias, P. Svarnas, N. Mourousias, K. Vratsinis & P. Vafeas, “**Generic residual charge based model for the interpretation of the electro–hydrodynamic effects in cold atmospheric pressure plasmas**”, *Plasma Sources Science and Technology (Plasma Sources Sci. Technol.)*, **28 (065005)**, 1–17 (2019).
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In Cold Atmospheric Pressure Plasmas (CAPPs), the residual charge that exists in the wake of the streamers plays an important role in the acceleration of the working gas. This paper presents a model that links the drift of the net residual ionic charge density, under the effect of the local electric field, with the momentum increase of the gas. In the model, the ions and the neutrals are considered as separate phases and the conservation equations for the two phases are connected via the ionic pressure. The residual charge density is quantified through an approximate approach that considers the streamer events to be “instantaneous”, in order to avoid the excessive computational cost of resolving the propagation of each streamer. For the validation of the residual charge model with the “instantaneous” streamer approach, comparisons are made with experimental data from three plasma jet reactors. The electrode configuration of the reactors and the varied parameters (applied voltage, gas flow rate) are chosen so as to cover a broad range of different cases, in order to assess the generality of the model. The comparisons concern the gas flow and visible plasma patterns. It is found that the numerically simulated flow structures are in agreement with the corresponding schlieren images and that the residual charge density is a fair indicator of the visible plasma channel.



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43. P. Vafeas, P. Bakalis & P.K. Papadopoulos, “**Effect of the magnetic field on the ferrofluid flow in a curved cylindrical annular duct**”, *Physics of Fluids (Phys. Fluids)*, **31 (117105)**, 1–15 (2019).
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The laminar fully developed ferrofluid flow of an otherwise magnetic fluid into a curved annular duct of circular cross-section, subjected to a transverse external magnetic field, is studied in the present work. The specific geometry is chosen as it is encountered in heat exchangers and mixers, where compactness is a priority. Results are obtained for different values of curvature, field strength and particles' volumetric concentration. A computational algorithm is used which couples the continuity, Navier–Stokes and magnetization equations, using a non–uniform grid. The velocity – pressure coupling is achieved using the so–called Continuity–Vorticity–Pressure (C.V.P.) variational equations method, adapted to the toroidal–poloidal coordinate system. The results confirm the ability of the method to produce accurate results in curvilinear coordinates and stretched grids, which is important for the standardization of the method's application to generalized coordinate systems. Concerning the micropolar flow characteristics, the results reveal the effect of the magnetic field on the ferrofluid flow. It is shown that the axial velocity distribution is highly affected by the field strength and the volumetric concentration, that the axial pressure drop depends almost linearly on the field strength and that a secondary flow is generated due to the combined effect of the external magnetic field and the curvature. The present analysis provides important insight into the effect of the three main parameters, revealing cases where a straight annular pipe might be preferable to a curved one and specific parts of the pipe that could be susceptible to enhanced loads, giving information that is crucial for design optimization.



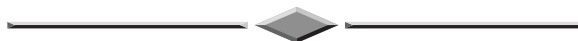
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44. G. Fragoyiannis, F. Kariotou & P. Vafeas, “**On the avascular ellipsoidal tumour growth model within a nutritive environment**”, *European Journal of Applied Mathematics (Eur. J. Appl. Math.)*, **31**, 111–142 (2020).
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The present work is part of a series of studies conducted by the authors on analytical models of avascular tumour growth that exhibit both geometrical anisotropy and physical inhomogeneity. In particular, we consider a tumour structure formed in distinct ellipsoidal regions occupied by cell populations at a certain stage of their biological cycle. The cancer cells receive nutrient by diffusion from an inhomogeneous supply and they are subject to an also inhomogeneous pressure field imposed by the tumour microenvironment. It is proved that the lack of symmetry is strongly connected to a special condition that should hold between the data imposed by the tumour’s surrounding, in order for the ellipsoidal growth to be realizable, a feature already present in other non-symmetrical yet more degenerate models. The nutrient and the inhibitor concentration, as well as the pressure field are provided in analytical fashion via closed form series solutions in terms of ellipsoidal eigenfunctions, while their behavior is demonstrated by indicative plots. The evolution equation of all the tumour’s ellipsoidal interfaces is postulated in ellipsoidal terms and a numerical implementation is provided in view of its solution. From the mathematical point of view, the ellipsoidal system is the most general coordinate system that the Laplace operator, which dominates the mathematical models of avascular growth, enjoys spectral decomposition. Therefore, we consider the ellipsoidal model presented in this work, as the most general analytic model describing the avascular growth in inhomogeneous environment. Additionally, due to the intrinsic degrees of freedom inherited to the model by the ellipsoidal geometry, the ellipsoidal model presented can be adapted to a very populous class of avascular tumours, varying in figure and in orientation.



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45. P. Vafeas, P.K. Papadopoulos, G.P. Vafakos, P. Svarnas & M. Doschoris, “**Modelling the electric field in reactors yielding cold atmospheric–pressure plasma jets**”, *Scientific Reports (Sci. Rep.)*, **10 (5694)**, 1–15 (2020).
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The behavior of the electric field in Cold Atmospheric–Pressure Plasma jets (CAPP jets) is important in many applications related to fundamental science and engineering, since it provides crucial information related to the characteristics of plasma. To this end, this study is focused on the analytic computation of the electric field in a standard plasma reactor system (in the absence of any space charge), considering the two principal configurations of either one–electrode or two–electrodes around a dielectric tube. The latter is considered of minor contribution to the field calculation that embodies the working gas, being an assumption for the current research. Our analytical technique employs the cylindrical geometry, properly adjusted to the plasma jet system, whereas handy subdomains separate the area of electric activity. Henceforth, we adapt the classical Maxwell’s potential theory for the calculation of the electric field, wherein standard Laplace’s equations are solved, supplemented by the appropriate boundary conditions and the limiting conduct at the exit of the nozzle. The theoretical approach matches the expected physics and captures the corresponding essential features in a fully three–dimensional fashion via the derivation of closed–form expressions for the related electrostatic fields as infinite series expansions of cylindrical harmonic eigenfunctions. The feasibility of our method for both cases of the described experimental setup is eventually demonstrated by efficiently incorporating the necessary numerical implementation of the obtained formulae. The analytical model is benchmarked against reported numerical results, whereas discrepancies are commented and prospective work is discussed.



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46. P. Vafeas, “**Low–frequency dipolar electromagnetic scattering by a solid ellipsoid in lossless environment**”, *Studies in Applied Mathematics (Stud. Appl. Math.)*, **145**, 217–246 (2020).
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Electromagnetic wave scattering phenomena for target identification are important in many applications related to fundamental science and engineering. Here, we present an analytical formulation for the calculation of the magnetic and electric fields that scatter off a highly conductive ellipsoidal body, located within an otherwise homogeneous and isotropic lossless medium. The primary excitation source assumes a time–harmonic magnetic dipole, precisely fixed and arbitrarily orientated that operates at low frequencies and produces the incident fields. The scattering problem itself is modeled with respect to rigorous expansions of the electromagnetic fields at the low–frequency regime in terms of positive integral powers of the real wave number of the ambient. Obviously, the Rayleigh static term and a few dynamic terms are sufficient for the purpose of the present work, since the additional terms are neglected due to their minor contribution. Therein, the classical Maxwell’s theory is suitably modified, leading to intertwined either Laplace’s or Poisson’s equations, accompanied by the impenetrable boundary conditions for the total fields and the limiting behavior at infinity. On the other hand, the complete spatial anisotropy of the three–dimensional space is secured via the introduction of the genuine ellipsoidal coordinate system, being appropriate for tackling incrementally such scattering boundary value problems. The non–axisymmetric fields are obtained via infinite series expansions in terms of ellipsoidal harmonic eigenfunctions, providing handy closed–form solutions in a compact fashion, whose validity is verified by a straightforward reduction to simpler geometries of the metal object. The main idea is to demonstrate an efficient methodology, according to which the constructed analytical formulae can offer the appropriate environment for a fast numerical estimation of the scattered electromagnetic fields that could be useful for real data inversion.

